

# PENTOMIND  



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Math Education.page

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## ACKNOWLEDGEMENTS

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## NOTES ON PENTOMINO ACTIVITIES

Pentominoes are the geometric shapes that are formed by joining five squares edge-to-edge. The name is derived from "domino," the shape obtained by joining two squares in this fashion. There are 12 distinct pentominoes, named after letters of the alphabet. (See drawing at the bottom of this page.) They can be put together to form many interesting shapes.
The puzzles in this book are self-explanatory. The instructions for all of them are the same: Cover all the figures using pentominoes. The pieces should not overlap or stick out beyond the boundaries of the figures.

The puzzles fall into six sets called Warm-Ups, Creatures, Pairs, Stairs, Rectangles, and Blow-Ups. As much as possible, the puzzles are ordered by difficulty within each set, the easier ones coming first.

The challenge of pentomino puzzles is to cover all the shapes on one page simultaneously with one set of pentominoes. To make the puzzles easier to solve, allow the students to borrow pieces from another set of pentominoes. Or, to make them even easier to solve, have the students cover one figure at a time. But, students have not really solved a puzzle until they can cover all of the figures on a page at the same time using pentominoes from a single set.

Students may solve the puzzles in any order, but if they are becoming frustrated, encourage them to work with the easier puzzles. Students will probably enjoy the puzzles more if they do not look at the solutions, but try to work out their own. Many of the puzzles are related. It may be helpful for students to keep a record of their solutions using graph paper. Then they can refer to a previous solution when working on a harder, related puzzle.

U


V

W




## THE PENTOMINO STORY

Mathematicians and puzzle buffs have enjoyed pentominoes since 1965 when Solomon Golomb, then a mathematician at the University of Southern California, wrote a book about them. Martin Gardner made pentominoes famous by writing several articles about them in SCIENTIFIC AMERICAN.

The best-known problem is to construct a 6 by 10 rectangle using all the pentominoes. Most humans have trouble finding even one solution, but a computer found 2,339 of them. How many can you find?

## EXPLANATION OF THE ACTIVITIES AND EXTRA CHALLENGES

Use pages 1 and 2 as patterns to make pentominoes or as a check to make certain you have a complete set.
Warm-Ups: Congruent pairs and triples. In geometry, figures are said to be congruent if they have exactly the same size and shape. Find other pairs or triples of congruent two-pentomino figures. It is possible to use all 12 pentominoes at once in making 3 such pairs. Note that all pairs shown on pages 5 through 11 consist of symmetric designs. Many more congruent pairs are possible if you do not require symmetry.

Creatures and Pairs: The solution to each of these puzzles is unique.
Stairs and Rectangles: Try to make pentomino rectangles or staircases that are larger than those shown in these sets. Make them in combination with smaller ones. Note that each rectangle on page 80 should have a 1 by 1 hole in it when the puzzle is solved.

Blow-Ups: In geometry, figures that have the same shape, but not necessarily the same size, are called similar.

Pages 82 and 83 show pentominoes and replicas with double the dimensions. How many pentominoes are needed to cover the double version?

Pages 84 and 85 are examples of triplication. How many pentominoes can be doubled? All can be tripled.

Pages 86,87 , and 88 show the doubling of two-pentomino figures. Can you solve them and use the remaining pentominoes to make a duplicate of the original two-pentomino figure? (This is called double duplication.)



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## CHECKLIST FOR THE PENTOMINO ACTIVITIES

Use this to keep track of which activities you have completed.

WARM-UPS
567891011121314151617

## CREATURES

181920212223242526272829303132

PAIRS
33343536373839404142434445464748495051

STAIRS
5253545556575859606162636465

## RECTANGLES

66676869707172737475767778798081

BLOW-UPS
82838485868788

Warm-Ups


Warm-Ups



## Warm-Ups




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## Warm-Ups




Warm-Ups



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Warm-Ups


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Creatures


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## Creatures



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## Pairs



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## Pairs



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Stairs



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Rectangles


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Rectangles


Rectangles



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## Rectangles

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Rectangles


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Rectangles

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## Solutions

You will learn the most from these solutions by not looking at them until you have solved the puzzles by yourself and recorded your solutions. Then compare your solutions to those given. (If they differ, please send the author a copy of your solutions.)

The solutions to figures that consist of only a few pentominoes are given by the letter names of the pieces involved. If you have forgotten the letter names of the pieces, refer to Notes.

Many puzzles involve two- and three-pentomino stairs, and $3 \times 5$ and $4 \times 5$ rectangles. The solutions to these four basic shapes are illustrated in Figures 1 through 4, and are referred to below by the names of the pieces used in each instance.
(Note that the following $4 \times 5$ rectangles are not used in the solutions mentioned above, and are therefore not illustrated: ILPV, ILTY, LPTV, LPUY, LPVY, LPVZ, LPWY, LPYZ, LUVY, NPUY.)


Fig. 1 Two-pentomino stairs


Fig. 2 Three-pentomino stairs


Fig. $3 \mathbf{3 \times 5}$ rectangles


Fig. $4 \mathbf{4 \times 5}$ rectangles

Congruent pairs:
5: PY, WX
7: NV, TU
9: IU, TZ
11: FL, TY
6: UY, vX
8: NP, UX
10: FT, IU

## Congruent triples:

| 12: $F Y, L Z, N V$ | 14: $L N, P T, W Z$ | 16: $F T, N Y, P Z$ |
| :--- | :--- | :--- |
| 13: $F P, L Y, N Z$ | 15: FP, NZ, TU | 17: LN, PU, VZ |

Three-pentomino stairs:

| 18: PYZ | 20: TWY | 22: ILW |
| :--- | :--- | :--- |
| 19: NVW | 21: IPY |  |

$3 \times 5$ rectangles:

| 23: LTY | 25: FPU | 27: PUV | 29: LNV |
| :--- | :--- | :--- | :--- |
| 24: LPV | 26: PUY | 28: NPU |  |
| $4 \times 5$ rectangles: |  |  |  |
| $30:$ FLUV | 31: NTVY | 32: PTVW |  |

Pairs of three-pentomino stairs:
33: IPY, NVW
34: ILW, PYZ
35: NVW, PYZ

Pairs of $3 \times 5$ rectangles:

| 36: LTY, NPU | 38: LNV, PUY | 40: LNV, FPU |  |
| :--- | :--- | :--- | :--- |
| 37: LTY, PUV 39: LTY, FPU   <br> Pairs of 4x5 rectangles:    <br> 41: FLPU, NTVY 44: FLTY, IPUV 47: FIPU, LNVZ 50: FLTY, PUVZ <br> 42: FPUY, ILNV 45: FPUY, LNVZ 48: IPUY, LNVZ 51: ILTY, PUVZ <br> 43: FLUY, PTVW 46: FIPU, NTVY 49: FLTY, INPU  |  |  |  |

52: LW, PY
55: IPY, LNV
53: ILW, PY
56: FW, IPUY
54: LNV, PY
57: FLPU, WTY

58:


59:




