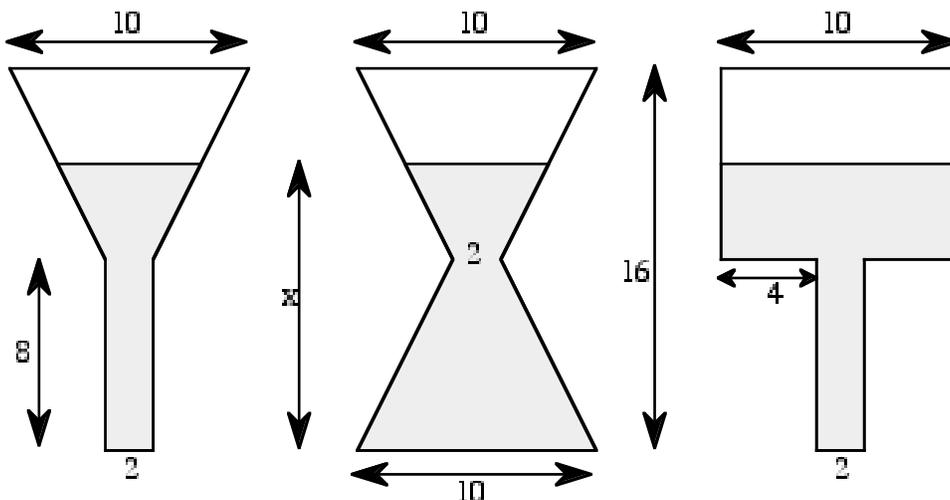


Doctor Dimension

Doctor Dimension is a flat scientist. He pours two-dimensional liquids into these two-dimensional containers:



- For each container, find a function for the amount of liquid (measured as area, since he lives in a flat universe) as a function of the height of liquid (x). Note that the functions are piecewise: each one consists of one part for $0 \leq x < 8$, and another for $8 \leq x \leq 16$.

To investigate the rate of change of these functions, you will need to set up your calculator as follows:

Program:

Press **PRGM**, choose NEW, and type the name RATE. Press **ENTER**.

Then, type:

```
:Prompt H
:Prompt X
:Disp "LEFT", (Y0(X) - Y0(X-H)) / H
```

To get "Prompt", press **PRGM**, then choose I/O. Likewise to get "Disp". Y_0 is in **VAR** Y-VARS. To complete the program, add the next line, which should give you the rate of change to the right of x . To use the program, press **PRGM**, then choose RATE.

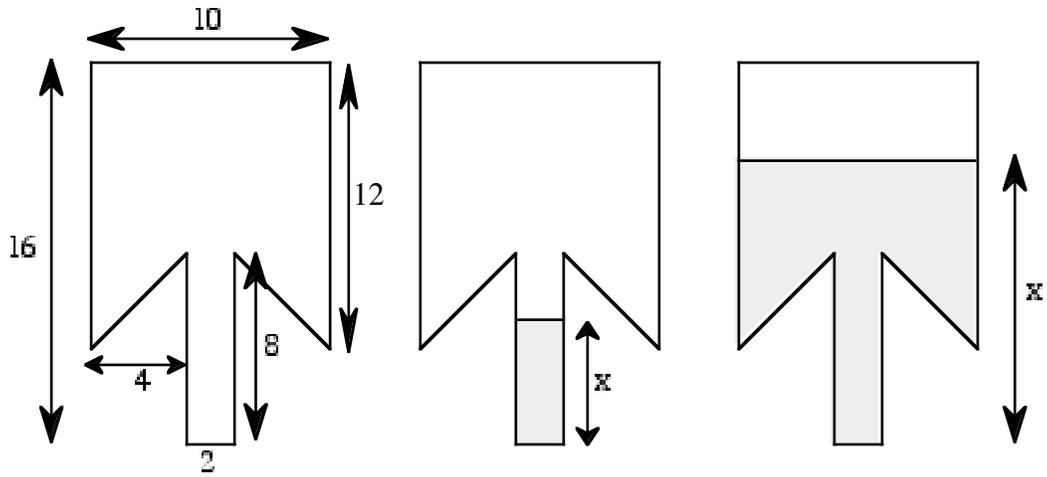
Function:

You can put the functions you want to study in any **Y=** place. To use the program on Y_1 , for example, go down to " $Y_0 =$ ", and set it to " $Y_0 = Y_1$ ".

- For each function above, use the RATE program to find its slope when $x = 4$, when $x = 8$, and when $x = 12$. Remember that to have a slope, the limits of the rates of change on the left and on the right must be equal.

The Return of Doctor Dimension

Doctor Dimension has a new two-dimensional container for his two-dimensional liquids:



He fills it starting in the middle, as shown in the figure.

1. Can you find a function for the amount of liquid (measured as area, since he lives in a flat universe) as a function of the height of liquid (x)?
2. Discuss what happens when $x = 8$.