

## Project: Other Bifurcation Graphs

Do a full orbit analysis of another “dome” function in the unit interval. Your function must be different from all other students’. Use the work we did on the logistic equation as a guide.

Possible functions:

Square root variations of the logistic function:  $y = r\sqrt{x(1-x)}$ ,  $r \cdot x\sqrt{1-x}$ ,  $r(1-x)\sqrt{x}$

Ellipse:  $y = r\sqrt{.25 - (x - .5)^2}$

Sine:  $y = r \sin(\pi x)$

Cubic:  $y = r \cdot x(x-1)(x-p)$ , with a specific  $p$  somewhere between 1 and 4

Fourth degree:  $r \cdot x^2(x-1)(x-p)$ , with a specific  $p$  somewhere between 1 and 4

1. For what value of  $r$  do we get extinction? equilibrium? cyclical behavior? chaos? (for each of these behaviors give a description and examples)
2. Where are the fixed points? How do they show up graphically? Do they attract or repel?
3. What are the points of bifurcation? (What does that mean?) Explain this in terms of the graph of the function  $f(f(x))$ , etc.
4. Do you see any other patterns? Any ways that your function differs from the logistic equation?
5. If you know calculus, try to bring your understanding of derivatives and slope into this.
6. Include a short introduction (or conclusion) based on the readings, about how this sort of analysis fits in the recent history of science. However, make sure the math is the main part of your paper.
7. If you can, include mathematical ideas from the reading, such as the use of  $\lambda$  as a parameter, or any of the other ideas you might pick up from a close re-reading.

Your report should include answers to the questions, but do not write it as a series of answers. Think of it as an article to bring this information to someone who has taken Math 3, but not Infinity. It should be written *and illustrated* clearly.