Algebra: Themes, Tools, Concepts

An exploration of a progressive curriculum
and some thoughts on the future of progressive math reforms

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Algebra: Themes, Tools, Concepts (ATTC) is a 508-page, progressive algebra curriculum. Developed by math educators Henri Picciotto and Anita Wah, ATTC is a response to the limitations of traditional Algebra 1 curriculums. When evaluated through a Deweyian lens, one can find several notable features of a progressive curriculum. The use of tools, a thematic approach, and an emphasis on group learning make for a rather revolutionary Algebra 1 program. In fact, the authors warn that that ATTC is not a “superficial remake of the traditional textbook;” employing it involves making a major leap towards math reform.\(^1\) While the potential of such a curriculum is grand, the realities of its success have been minimal. When implemented by the right teacher in the right context, ATTC makes algebra come alive for students, promoting deep understanding of mathematical concepts. But in the hands of the average teacher and school, ATTC is a daunting directive. As is the case with many progressive curriculums, ATTC has been passed over for the more traditional programs. This makes one wonder: What is it about progressive curriculums that make implementation so difficult? Why are the traditional curriculums more attractive? What does this mean for the future of progressive math curriculums? After an explanation of the curriculum and a look at its progressive features, I will attempt to address these questions.

A brief description of ATTC

Before describing the curriculum, I should mention my relationship to it. First, in 1993, as a second year teacher, I took a workshop with Picciotto at the Urban School. After the workshop, I purchased the texts and returned to my Ohio classroom where I employed aspects of the curriculum. Several years later (summer 2001), Picciotto and I conducted an intensive 3-week Algebra Institute for Bay Area teachers. To accomplish our goal of “helping teachers achieve a deeper understanding of the math they teach,” we used several lessons and activities from ATTC. Both in my classroom and in the teacher workshops, I have witnessed the amazing learning that results from the activities in ATTC.

Picciotto and Wah classify themselves as math reformers and their curriculum certainly supports that title. Inspired by their own experiences as teachers and guided by the NCTM standards, Picciotto and Wah developed ATTC as an answer to the limitations of traditional algebra programs. Those limitations include:

1. One-dimensionality. (Traditional emphasis on the abstract manipulation of symbols excludes many students and is only a small part of what math is about.)

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2. Teacher as authoritarian, student as vessel to be filled.
3. Lack of relevance to both real-world situations and higher level math.
4. Skill and drill, at the expense of deep mathematical thinking.
5. Self-contained and discrete presentation of topics.²

This list of what the authors don’t like about traditional courses is a good springboard for looking more closely at the educational philosophies and methods in their text.

The ATTC curriculum is experiential and student-centered. The authors make several explicit statements about the importance of this. One such example follows:

“Most students will not remember concepts if they are explained once or twice by a teacher and practiced in isolation over a short period of time. Students must be involved in their own learning and have experiences with ideas in many forms and formats over an extended period of time. They must experiment, conjecture, discuss, and write about what they are thinking.” ³

One of the ways the curriculum is able to engage the students in their own learning is through the use of tools. There are four categories of tools: grid tools (graph paper, geoboards, tiles); visualization tools (Lab Gear®, Cartesian graph paper, function diagrams); computational tools (calculators and graphers); and pencil-and-paper tools (table of values). The tools enable students to be actively involved in mathematical exploration, constructing meaning for themselves. Because they are concrete, tools also allow for mathematical discussion of concepts that are often difficult to talk about in an abstract or symbolic manner.

The curriculum is not set up sequentially, but thematically. This thematic approach allows for connections to be made between various concepts in algebra, as well as other branches of math. For example, within the theme of “Area” students use a variety of tools to explore the following: the relationship of perimeter and area (leading to patterns and functions); the area of shapes on a geoboard (leading to development of formulas, geometric reasoning, and square roots); and area with the Lab Gear (leading to the distributive property, trinomial patterns, and factoring). In ATTC, “Area” becomes the context for exploring several mathematical concepts instead of simply a discrete chapter of math formulas. This thematic approach also allows for spiraling. A concept can be introduced, explored, and revisited across themes.

The authors believe that while “symbol manipulation is a useful tool, accurate and/or speedy manipulation is no longer defensible as a central goal of the new algebra.” ⁴ With that in mind, the

³ Ibid., T6.
⁴ Ibid., T19.
goal of the ATTC curriculum is to use a multi-dimensional approach that will provide students with a deep understanding of algebra concepts and their relation to one another. Tools and themes are “the means, not the end” to this kind of understanding. Tools and themes help create an environment where students are interested and engaged, where they experience mathematical ideas for themselves, and where they come to understand the structure of mathematics.

The authors understand that tools and themes can’t do it all. In order for the curriculum to be effective, there must be significant pedagogical inspiration. The teacher’s guide explicitly states that using ATTC will require a different kind of teaching and provides numerous pedagogical tips to that end. Teachers must learn to guide, not dictate. They should promote questioning, risk-taking and reflection. Other strategies for implementing ATTC include the use of cooperative group work, whole class discussions, writing, and a range of assessments.

Just like the curriculum itself, assessment in ATTC is non-traditional. Focus is placed on the intrinsic rewards of learning. To that end, there are evaluations on class discussion, group work, homework, and notebook organization that are judged solely on effort. The authors support formal assessment tools (tests and quizzes), but say that they should be accompanied by reports and projects. Portfolios would provide the ultimate device for student and teacher to track understanding and progress. There is also a recommendation to move away from grades (report cards) and towards a system of narratives.

A visual representation of the ATTC approach to Algebra can be seen below. Tools and themes support the development of concepts. Note that skills are acquired within a framework of problem solving, not separate from the exploration of math. Also, observe that these explorations (problem solving) are the jumping off point of the math experience, not simply the end point of application.

Ibid., T22.
Looking at ATTC through a Deweyian lens.

The ATTC curriculum would receive solid marks from Dewey. For starters, he would applaud the authors’ response to the “artificial dualism” of student and the curriculum. The curriculum cannot be considered without the student, and vice-versa. Picciotto and Wah developed ATTC while working with their own students at the Urban School. Much like Dewey’s Lab School, their classrooms were the place to test out ideas and strategies. This model of teacher-as-researcher allowed for a better understanding of student interest and comprehension, and in turn, an improvement of craft and curriculum.

Dewey’s idea of a meaningful educational experience is well represented in ATTC. Dewey believes that educational experiences must be both continuous and interactive. For an experience to be continuous, it must stimulate curiosity, create purpose, and promote further learning. This is the longitudinal aspect of the educational experience. The lateral aspect, interaction, is the relationship between the internal conditions of the individual and the external conditions of the environment. It is the responsibility of the educator to provide for both: know where a student is at, engage them, and create an environment where they can grow. Although ATTC does not explicitly tell teachers how to determine the mathematical interests of students, it does state that the mathematics should be interesting to the students. Activities such as “Math on Another Planet” and “Mexican Food” are meant to engage students, at the same time that they build math knowledge. As the curriculum proceeds, understanding isn’t random, it is connected and expanding. ATTC also does a nice job of attending to issues of interaction through what Dewey refers to as “objective conditions.” Teacher-as-facilitator, tools, group work, and math games are just a few of the ways that ATTC is able to create a positive learning situation.

Dewey would appreciate ATTC’s attempt to move away from the sequential, discrete classification of math that is prevalent in the traditional texts. For Dewey, a subdivided, unit-based curriculum often provides students with mis-educative experience - experiences that lack meaning, stunt growth, and leave students with a “who cares?” attitude. Picciotto would agree. Doling out isolated bits of math for students to assimilate and memorize does not result in mathematical learning. He goes on, in quite a Deweyian fashion to state: “[Traditional texts] sabotage the students’ own engagement with the material, as they get the idea that math is about reading how

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7 Ibid., 25-26.
somebody else summarizes something that they feel they themselves could never aspire to understand.”

A student’s experience with math shouldn’t be imposed from outside; it should be nurtured from within. To that end, the progressive math classroom should be a place where ideas are discovered, discussed, challenged, and rebuilt. ATTC certainly attempts to provide such an experience.

Although the math problems in ATTC do not deal specifically with social issues, there is certainly a social agenda behind the curriculum. Picciotto realizes that Algebra is a gatekeeping subject, one that determines whether students have access to higher learning opportunities, as well as college. He feels that providing all students access through this gate should be a primary goal of any algebra curriculum. He also believes that watering down the traditional text is not the way to do this. As a result, ATTC seeks to offer a rigorous algebra experience for a wide range of students, with the intention of giving more students opportunities for math and science beyond Algebra. Indeed, this feature of educational equity is one with which Dewey would be impressed.

The final progressive feature of the curriculum lies in the role of the teacher. In the eyes of both Picciotto and Dewey, the teacher is clearly the conductor of a complex symphony of learning. The math knowledge does not flow from the teacher, it is nurtured in the student. The teacher must act as facilitator and guide, choosing and arranging activities to create meaningful educative experiences. As Dewey says, planning must be “flexible enough to permit free play for individuality of experience and yet firm enough to give direction towards continuous development of power.”

This is clearly the one of the goals of ATTC curriculum; however, it is a grand request. In order for the teacher to “conduct this symphony,” she must possess both a knowledge of her students and a deep understanding of the mathematics. From this view, ATTC is clearly not a text for the amateur or uninterested math teacher. ATTC calls for the most progressive of educators.

**ATTC in practice**

ATTC was published in 1994. Around the time of publication, Picciotto and Wah conducted several workshops to promote the curriculum. ATTC was adopted in a few districts, but unfortunately, its success was limited. For the most part, teachers found the curriculum “too difficult use.” According to Picciotto, the schools that still use ATTC are mostly private. He does

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know of several teachers, both public and private, who use activities from the text to supplement their traditional curriculum.

ATTC is still the primary algebra text used at the Urban School, where Picciotto continues to teach. The text and tools are successfully used by all Urban math teachers, not just Picciotto.

**Limitations of ATTC**

Like most curriculums, ATTC has its limitations. The feedback that Picciotto has received indicates that these limitations appear to be issues of design rather than substance. First of all the curriculum is huge, not just in size (508 pages!), but in content. There is no way a class could “cover” the text in one year. This might be resolved by having a teacher pick and choose the most important algebra activities, but the non-sequential set-up of ATTC impedes this. Because algebra concepts are integrated across themes, it is hard to locate exactly where the concept is developed. For example, functions are explored in almost every theme, making it hard to pull out the “critical” activities on functions. Coverage concerns also apply to the daily lessons. The explorations take time; tool use, experimentation, and discussion cannot be hurried along. Most of the activities simply could not be conducted in a 45-minute traditional class. In addition, to the frustrations of time, many teachers found the group work difficult to manage. Because students are exploring at various rates, teachers felt it was hard to “keep kids on track.”

**Demands of ATTC**

For certain there are limitations to ATTC, ones that both Picciotto and Wah acknowledge. But I believe that there is another way to look at some of these limitations. As someone who has used the curriculum (and as an ever evolving progressive educator!), I think that most of challenges to implementing ATTC have to do with its progressive nature, and in turn, the demands such a curriculum place on the teacher and the classroom. The assumption that this text makes is that the teacher has a phenomenal understanding of mathematical concepts, as well as, a high comfort level with innovative pedagogy. The teacher of ATTC must be somewhat of “mathematics revolutionary.” She must believe in the benefits of an integrated curriculum. She must be willing to transfer the locus of control for the learning from herself to the student. Whenever possible, she must guide the students to participate in their own learning, while still making sure that their mathematical experience is deep and meaningful. In effect, any teacher who uses ATTC must leave behind their traditional ways and fully commit to a progressive pedagogy.
Looking at how ATTC is employed at the Urban School, one can see other features that would be necessary for the successful implementation of such a curriculum. At Urban, classes are divided into 70 minute blocks. This allows plenty of time for the math explorations, group work and reflection. Teachers at Urban don’t work in isolation, they collaborate. Weekly department meetings allow teachers to discuss the lessons, anticipate problems, share strategies, and in general, engage in rich mathematical conversation. Other features of the Urban School that facilitate the implementation of ATTC include the use of narratives rather than grades and an emphasis on cooperative learning.

Although limited, my personal experience with ATTC has been inspiring. Both in my classroom and when working with teachers, I have been amazed by the profound math learning that ATTC promotes. When ATTC is employed during our teacher workshops, the feedback is fantastic. Many teachers say they “see” the math for the first time. They enjoy the diverse algebraic experiences that the themes and tools provide. Most teachers say they leave these workshops with a significantly deeper understanding of Algebra. But when asked why they don’t use ATTC in their classrooms, the responses are framed through a traditional lens: “Too complicated.” “Not enough time.” “How would I ever manage the group work and all those tools?” “I can’t teach math thematically.” “I need to make sure I cover the material.” How is it that these teachers can feel the power and potential of a progressive curriculum, and yet, reject the idea of employing such a curriculum?

**The demands of a progressive curriculum**

ATTC is not the first progressive curriculum to face challenges of implementation. Progressive education is difficult to put into practice. Dewey himself never had any illusions about the “ease” of implementing progressive education. In *Education and Experience*, when commenting on such reform, he states: “the process is slow and arduous. It is a matter of growth and there are many obstacles which tend to obstruct growth and deflect it into wrong lines.” 10 Little did Dewey know all the various roads and obstacles progressive education would encounter during its long journey through the 20th century.

The history of progressive education over the past eight decades is an interesting one, but it is a topic for another paper. Suffice it to say, the “Traditional” vs. “Progressive” battle has had

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many matches, and for most of the twentieth century, “Traditional” has been wearing the victory belt. For certain, there are some very complex issues to consider when examining why the progressive education movement never fully took root. Multiple definitions of the term “progressive,” demands of a growing and diverse population, difficulty determining “measurable outcomes,” and an ongoing debate over the purpose of schools are but a few of the myriad of issues that challenged sustainable progressive education. But probably one of the most critical considerations for progressive reform has been the role of the teacher.

The significance of the teacher in the realization of a progressive curriculum cannot be underestimated. In the progressive system, teachers are not merely detached observers, (as they were often mistaken to be); rather, they are choreographers who have the difficult task of creating a magnificent educational dance. Dewey knew that what he envisioned for schools would place a “tax upon the educator,” and he realized these demands would be far greater for the progressive teacher than the traditional one. 11 Consider for a moment, how a job description for a teacher at the Lab School might read:

**Math educator wanted.** Must be energetic, thoughtful, and creative. Must know subject matter deeply. Must be able to develop a meaningful math experience around the interests of the children. Must provide activities that allow students to construct their own understanding of math concepts, while building a rich understanding of the relevance and connectedness of mathematics. Must use multiple means to achieve this. Must promote questioning and reflection. Must use cooperative learning and group discussions whenever appropriate. Must believe in the education of all for the purpose of democracy.

How many educators today would apply for such a job? This revolutionary vision of teaching is a inspirational one, but it is one that caused difficulty for the progressive movement. In his book, The Genius of American Education, Lawrence Cremin underscores this fact when he says: “progressive education demanded infinitely skilled teachers and it failed because such teachers could not be recruited in sufficient numbers.” 12 Cremin’s writings point to a number of other obstacles to the progressive movement, but the lack of talented, creative teachers was a significant one.

Context is also crucial for the implementation of a progressive curriculum. Dewey was adamant that traditional structures of desks-in-a-row and regimented schedules were not conducive to meaningful learning. Just as we might imagine a job description for a teacher, we can also envision what a progressive school looks like. The “Dewey Schools” presentations provided us with a wonderful image of such a place: open classrooms with large spaces for presenting and

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11 Dewey, Experience and Education, 40.
performing; small class sizes to allow for close relationships between students and teachers; time for teachers to meet and collaborate; flexible schedules; performance based assessments; and integrated, discovery-based teaching. Several of these features map directly to The Urban School, where the ATTC curriculum continues to thrive. The fact that The Urban School is private is not incongruous with the reality of successful progressive schools during the twentieth century. In their study of such schools, Semel and Sadovnik point to the fact that many of the institutions where progressive ideals flourished were in fact small, private schools. The systems and structures at these schools could easily be adjusted to accommodate a progressive curriculum. Thinking about how to achieve this on a larger scale is daunting.

The trappings of the traditional track

It is clear that the teacher and context matter in the success of a progressive curriculum. Does this mean that progressive programs are limited to private schools? Despite the middle and upper class nature of early progressive schools, Dewey’s educational vision was not meant for the elite few. But again, Dewey had no illusions about the challenges facing teachers and schools that wanted to move towards a progressive program. For teachers of mathematics, that move has been near to impossible. In the twentieth century, the dominant method of teaching mathematics has been through the teacher-centered delivery of fragmented parcels of math content. This method, largely influenced by Thorndike and Skinner, is based on the idea that behavior is directly related to external stimulus; that is, people will do what they are reinforced to do. Learning then becomes the “acquisition of very specific skills and bits of knowledge, a process that is linear, incremental, and measurable.” We know how this looks in the math classroom: teacher in front telling, students in rows receiving, and frequent tests to evaluate progress. This method is also reinforced by the external demands places on the school and the teacher. When agendas of coverage and standardized tests are constrained by class size and inflexible schedules, the stand and deliver method seems to be the only viable solution. For many of the teachers in my summer workshop, the idea of “turning the learning over to the child” sounded like a recipe for disaster.

Progressive math classrooms are so different than traditional ones that people who only know the latter, wonder how the former could possibly be delivering math. This is a valid concern.

13 Susan Semel, Schools of Tommorrow, Schools of Today, (New York: Peter Lang, 1999), 20.
In some instances, math reforms have been implemented in the name of equity and progressivism, only to result in a watering down of the traditional programs.\textsuperscript{15} Not surprisingly, the reaction from the public is often a “Back-to-basics” cheer, calling for more math (usually skill) and more testing (usually drill). The idea is that children must take their math like they take their medicine. As Dewey says when speaking about the child in the traditional classroom, “[he] is simply the immature being who is to be matured; he is the superficial being who is to be deepened; his is narrow experience to be widened. It is his to receive, to accept. His part is fulfilled when he is ductile and docile.” (p.186) This “vessel to be filled” analogy is a far cry from the progressive view of the student.

The image above is a pathetic one, and yet it is one that prevails in most American math classrooms. The belief is that the traditional classroom and teacher will deliver the most math. This is best done by implementing “teacher-proof” curriculums that require little thinking about mathematics and teaching. The irony here is that we know the traditional system only works for a very small percentage of students, and overall, does not produce the kinds of mathematical thinkers we want our students to be.\textsuperscript{16} So how do we help shift the thinking about teaching in mathematics away from the traditional camp and towards a more progressive one?

What is the future of ATTC and other progressive math curriculums?

Like the Dark Side, the force of the Traditional is strong. Shifting such a mindset will be challenging. Despite this, I see a hopeful future for progressive curriculums and teaching. First, there is the simple fact that people know the difference between a meaningful educative experience and a bad one. Student-centered learning feels good! The teachers in our summer workshop openly expressed how excited they were about the math they were learning. There is no question that this was directly due to the fact that they were actively engaged in constructing knowledge for themselves. Second, the research on thinking and learning in mathematics points to the power of discovery based learning. In his article, “The Mathematical Miseducation of America’s Youth,” Michael Battista states “all current major scientific theories describing students’ mathematics learning agree that mathematical ideas must be personally constructed by students as they try to make sense of situations.” \textsuperscript{17}

\textsuperscript{15} Henri Picciotto, http://www.picciotto.org/math-ed/reply-to-critic.html
\textsuperscript{17} Battista, \textit{Phi Delta Kappan}, 429.
Probably the most influential factor that supports the move away from the traditional methods, comes from the NCTM Standards. Produced by mathematicians, mathematics educators, and researchers, the Standards represent a consensus among experts in field, about what should be taught and learned in mathematics. An emphasis is placed on the attainment of “mathematical power” defined as the ability to “explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems.” The implications for pedagogy are evident. In her essay on “Transforming Mathematics Education,” Martha Stone Wiske notes that to produce mathematically literate students, teachers and schools must move away from the “transmission-of-information” paradigm to the “construction-of-understanding” paradigm. Wiske goes on to say that such a paradigm shift will require fundamental changes in the beliefs and knowledge that teachers possess. In addition, time, materials, and the necessary organizational support will be required to facilitate such a transition.

The recommendations the Standards make for the future of mathematics teaching are grand. However, the soundness of the document and the credibility of the authors have resulted in a large scale embracing of such changes. But we must remember that change happens slowly. ATTC was developed with the Standards in mind, and yet, its adoption has been limited. Despite this fact, I know that the teachers in our summer workshop made some incremental changes to more progressive teaching methods. Their beliefs about mathematics learning are slowly shifting, and in turn, so is their teaching. I am frustrated with the slow pace of such change, but it is better than the status quo. When I recently spoke with Picciotto about my concern for the future of mathematics teaching, he urged me to focus on balance. He said “the problem with reforms is that they swing hard one way or the other, without considering the benefits of both sides.” He went on to say that there is too much emphasis on the extremes: traditional vs. progressive; skills vs. understanding; instruction vs. discovery. This “versus” mentality will only impede progressive reforms; we have to find ways to blend both. Again, I think Dewey would be proud.

I think in the end we have to decide why it is we teach mathematics. If it is because we covet the power that comes with standing in front of a room, then maybe the traditional method will always have the upper hand. But I believe, that our reasons for teaching math are more profound that that. I know for myself, teaching is about empowering young people to realize their potential as doers and thinkers of mathematics. For me, like Dewey, “the child is the starting point, the

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center and the end. His development, his growth, is the ideal.” I believe that this philosophy must be a part of any curriculum development in mathematics. The future of a student-centered curriculum like ATTC may have already been decided, but I am confident that other progressive curriculums are on the way.

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