

Three Quadratic Forms

Only use an electronic grapher to check your answers!

Factored Form

1. For the function $y = 2(x + 3)(x - 4)$:
 - a. What are the *roots* (the x-intercepts)? (Hint: what is the value of y at the x-intercepts?)
 - b. What is the y-intercept?
 - c. What is the x-coordinate of the vertex?
 - d. What is the y-coordinate of the vertex?

The following questions are about *factored form*: $y = a(x - p)(x - q)$

2. How do we know p and q are the roots?
3. Find, in terms of a, p, and q:
 - a. The y-intercept.
 - b. The x-coordinate of the vertex. (Hint: it's half way between the roots)
 - c. **Challenge**: The y-coordinate of the vertex.

From Factored Form to Standard Form

$y = ax^2 + bx + c$ is called *standard form*. Some of the information that is obvious in factored form is not as visible in standard form.

4. Take the equation $y = a(x - p)(x - q)$, and distribute, so as to write it in standard form. [Hint: this is a two-step process. One way to do it is to first multiply $a(x - p)$. Then multiply the product by $(x - q)$.]
5. Write formulas for b and c in terms of a, p, and q. The trickiest part is to figure out b — it requires factoring the x.
6. Find, in terms of a, b, and c:
 - a. The y-intercept.
 - b. The sum of the roots.
 - c. The product of the roots.
 - d. The x-coordinate of the vertex.
 - e. **Challenge**: The y-coordinate of the vertex.

Note that the x-coordinate of the vertex does not depend on c. It follows that the formula still works even if the function has no roots: by changing c, we can change it to a function that does have roots without affecting the x-coordinate of the vertex.

Vertex Form

7. Consider the expression $2(x - 3)^2 + 4$
 - a. What is the smallest value it can possibly be? Explain, without referring to a graph.
 - b. For what value of x does it reach this smallest value? Explain, without referring to a graph.
8. Consider the expression $a(x - h)^2 + v$, with $a > 0$
 - a. What is the smallest value it can possibly be? Explain.
 - b. For what value of x does it reach this smallest value? Explain.
 - c. What happens if $a < 0$?

It follows that (h, v) are the coordinates of the vertex of the parabola with equation $y = a(x - h)^2 + v$. This is *vertex form*.

From Vertex Form to Standard Form

Some of the information that is obvious in vertex form is not as visible in standard form.

9. Take the equation $y = a(x - h)^2 + v$, and distribute, so as to write it in standard form.
10. Write formulas for b and c in terms of a , h , and v .
11. Find h as a function of a and b .
12. **Challenge:** Find v as a function of a , b , and c .

Are There Roots?

The y -coordinate of the vertex, which you might have found in #6e or 12 is $v = \frac{-b^2 + 4ac}{4a}$.

13. If you got a different answer, figure out whether you made a mistake, or whether your solution was equivalent to this one.

We will use a and v to determine if a quadratic function has roots.

14. Since $a \neq 0$, there are two cases:
 - a. If $a > 0$, we have a “smile” parabola. In this case, the parabola intersects the x -axis if _____
 - b. If $a < 0$, we have a “frown” parabola. In this case, the parabola intersects the x -axis if _____
15. **Challenge:** Find an inequality involving only a , b , and c that one can use to figure out whether the parabola intersects the x -axis. (Hint: It can be deduced from the two cases above and the formula for v .)