Scaling Teacher Notes

This packet is about scaling, including scale drawings. It is of course related to proportional relationships, for which it provides a visual context. This will serve both as a review and as an opportunity to get deeper understanding by looking at a different representation.

Because of the new context, the unit starts with examples that are numerically straightforward, and moves towards "real world" numbers, which are more difficult to work with.

Pentominoes

The unit starts with pages 52-53 from *Working with Pentominoes*. Note that since the book comes with an e-book version on DVD, you can project these pages, and use virtual pentominoes on your computer screen.

Similarity (page 52)

The main point is to get across that to scale a picture you need to multiply *all* its dimensions by the same number. Let the students know that they will have to scale figures themselves in the next lesson, for which they will need the ideas from this lesson.

The worksheet includes questions about area. It is not important at this stage for students to see the pattern there. All they need is to realize that the scaling factor applies to the dimensions of the sides, not to the areas. The most important questions are #1, 2, and 8. #9-10 are definitely not required.

Spend a few minutes discussing the note at the bottom of the page, not necessarily to reach resolution on the issue but to focus students' attention on it so they know it is important. You might want to open the discussion by projecting the two figures but covering the Note, and taking a vote: who thinks these two figures are similar and who thinks they are not. You could then have someone with each position make their case, or ask a volunteer to measure the side lengths and then report their findings. You might also ask students for sketches (on graph paper) of two other figures that look similar but aren't.

Doubling (page 53)

Important: the lesson is not entirely on page 53. Please read on for the full lesson plan.

On the surface, the goal of this lesson is to create and solve pentomino puzzles. The key mathematical moment is when students create their own scaled pentominoes because to do this, students must realize that *every* side length needs to be scaled. This is why for this unit, I do not recommend using pages 54-56.

When you photocopy page 53, make sure the image of the F pentomino has a base that measures 2 inches. Make a test copy, and if it's off, make the needed adjustments.

You will need one-inch graph paper, which is available on page 95 of the book. Be similarly careful with the size of the squares in the photocopies.

Here is the plan for the lesson:

- Hand out the pentominoes, as much as possible by giving neighbors different-colored sets.
- Hand out the worksheet, and tell the students to cover the scaled F pentomino with pentominoes from their set. This is quite challenging, and you should not expect the whole class to solve it. As soon as a few students have answers, you can share them on a document camera, or using the *Working with Pentominoes* e-book.
- Show the students how to record their solution to the puzzle by tracing the pentominoes one by one, and optionally coloring the result. You may also record the solution on the smaller "F" at the bottom of the page, but that is not essential.
- Hand out one-inch graph paper, and tell the students they should create their own puzzles by doubling or tripling the pentomino shapes. Emphasize that *all* dimensions must be doubled or tripled. (That is the main point of this lesson.) Help students as they create the puzzles.
- Each student should try to solve the puzzle they created, record the solution by tracing and possibly coloring the shapes, and move on to make another puzzle.

In the case of tripling, the only pentominoes that will fit on one piece of paper are the P and the U. For the others, you will need to tape two pieces of one-inch graph paper together.

While all the tripled pentominoes can be solved as puzzles, they are of course more difficult than the doubled pentominoes. On the other hand, note that not all doubled pentominoes can be solved. (The V and the X are impossible.)

Solved and colored puzzles make a great bulletin board display.

Scaling

This page introduces what is perhaps the most common use of scale drawings: a map. In order to keep calculations manageable, I chose a scaling factor of 2,000,000.

For the lesson to work at all, it is crucial that you verify that the Monterey to San Jose distance on the photocopies you give the students is indeed 4.2 cm. Make a test copy, and if needed adjust the size.

Of course, students will need centimeter rulers to do this work.

You might point out that the calculations needed to fill out the table can be made mentally, by giving Monterey to San Jose as an example. Ask students for their ideas of how this can be done. It is definitely a legitimate idea to first calculate the real distance in kilometers, which can be done by multiplying the map distance by 20.

Because multiplication is easier than division, it is best to fill out the table before attempting to answer #3. Hold a class discussion after #3, and make sure students write down their answers to #4 before the end of the period.

Stuart Little

This lesson and the next two are based on *Algebra: Themes, Tools, Concepts* 6.11.

In this lesson, students will draw scale models based on real world measurements. You will need tape measures and rulers.

Do not spend an enormous amount of time reviewing or discussing how to calculate the average, as it is not a central point of the lesson. After a few boys have been measured, tell the students how to quickly calculate the average.

Tell the students that to keep the remaining calculations manageable, you will round this number to the nearest inch, or if you're not too far from a multiple of 10, to the nearest multiple of 10.

Obviously, it is debatable how to define "length and width" for items of clothing. You should have some class discussion of this, and remind the students that they will be making 2-dimensional scale drawings of these, so that measurements like the length around the waist are not relevant. They should picture the items of clothing laid flat on a table, and figure out what those measurements would be.

#4, of course, is the heart of the mathematics for this lesson. If students have trouble, resist the temptation to just tell them what to do before they even try. Help them by referring to the activities of the past few days: how did we get the map distance from the real distance? How did we get the side lengths of a scaled pentomino? If necessary, of course, give the procedure to students who really need it, but warn them that there will be a similar activity the next day.

Alice

This lesson is very much like the preceding one, except that Alice first shrinks, then expands. In the latter case, it is not possible to make actual-size drawings.

Again, do not spend an enormous amount of time reviewing or discussing how to calculate the average, as it is not a central point of the lesson. After a few girls have been measured, tell the students how to quickly calculate the average.

Tell the students that to keep the remaining calculations manageable, you will round this number to the nearest inch, or if you're not too far from a multiple of 10, to the nearest multiple of 10.

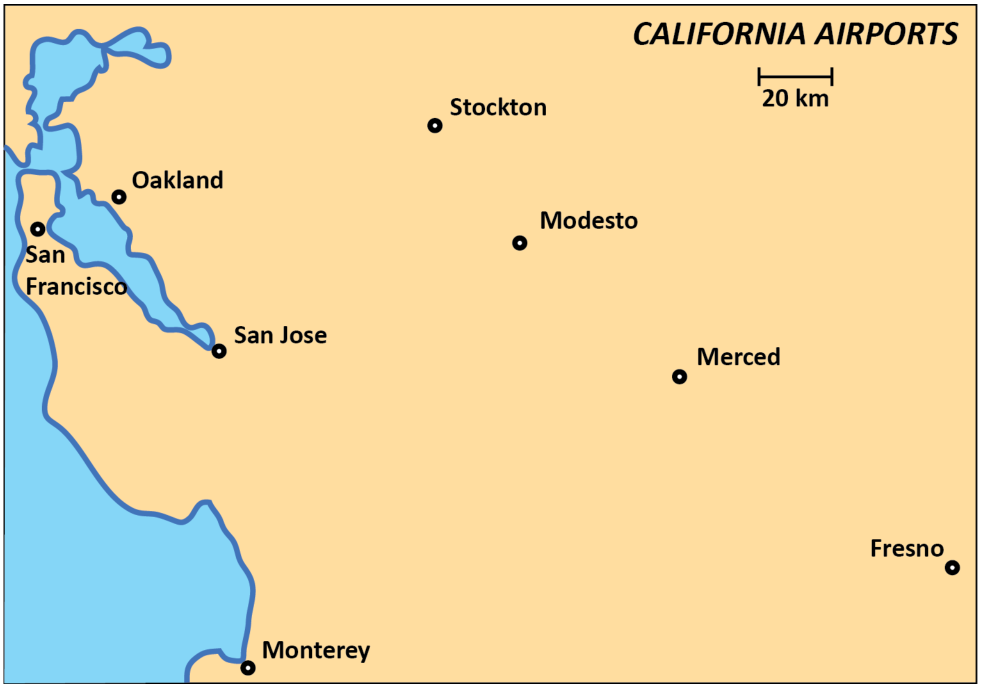
For #2, ask students to use calculators and trial and error to find a scaling factor they can use to multiply the height you found in #1 in order to get approximately 10 inches. Of course, the number will be less than 1.

Scaling Projects

Students can choose one of the two creative projects. These can serve as an assessment of how well they learned the key ideas about scaling.

The research project might work better as a project for the whole class, with help from the teacher, because it involves not only the math of this unit, but also requires some real-world research.

Scaling



This map shows eight California airports. The distance between Monterey and San Jose on the map is approximately 4.2 centimeters. In reality, it is approximately 84 kilometers, which is the same as 8,400,000 centimeters.

1. The real distance is how many times the map distance?

The map distance and the real distance are in a proportional relationship. The number you found in problem 1 is called the *scaling factor*. It is the number by which you must multiply the map distance to get the real distance.

2. Fill out this table with approximate measurements:

	Map Distance (cm)	Real Distance (cm)	Real Distance (km)
Monterey to San Jose	4.2	8,400,000	84
San Jose to Oakland			
Oakland to Stockton			
Stockton to Modesto			
Modesto to Merced			
Merced to Fresno			

3. The distance from Fresno to Monterey is approximately 188 kilometers.
   a. Without measuring, predict what it should be on the map.

   b. Check your prediction.

4. Complete these sentences:
   a. To get the real distance from the map distance, you must ____________________________

   b. To get the map distance from the real distance, you must ____________________________

Stuart Little

Here is the beginning of *Stuart Little,* a children's book by E.B. White.

> When Mrs. Frederick C. Little's second son arrived, everybody noticed that he was not much bigger than a mouse. The truth of the matter was, the baby looked very much like a mouse in every way. He was only about two inches high; and he had a mouse's sharp nose, a mouse's tail, a mouse's whiskers. and the pleasant, shy manner of a mouse. Before he was many days old he was not only looking like a mouse but acting like one, too —wearing a gray hat and carrying a small cane. Mr. and Mrs. Little named him Stuart, and Mr. Little made him a tiny bed out of four clothespins and a cigarette box.

You will draw clothes and school items at the actual size they should be for Stuart Little. The first thing you will need is a scaling factor.

1. Measure, in inches, the height of several boys in your class who volunteer to be measured. Calculate the average height of your sample.

2. For a boy whose height is close to the average you found in problem 1, measure, in inches, the length and width of his:
   a. pants

   b. shirt or coat

3. Measure, in inches, the length and width of:
   a. a book or binder

   b. a chair or desk

4. Calculate the length and width of each item in problems 2-3, if it were to be made for Stuart Little. Write an explanation of your calculation.

5. Draw each item in the size that you calculated in problem 4.

Alice

Here is an excerpt from *Alice in Wonderland,* a book by the English mathematician Lewis Carroll.

…this bottle was not marked "poison," so Alice ventured to taste it, and finding it very nice, (it had, to fact, a sort of mixed flavour of cherry-tart, custard, pine-apple, roast turkey, toffee, and hot buttered toast), she very soon finished it off.

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"What a curious feeling!" said Alice. "I must be shutting up like a telescope!"

And so it was indeed: she was now only ten inches high, and her face brightened up at the thought that she was now the right size for going through the little door into that lovely garden.

You will find the size of objects that would be the appropriate size for a 10-inch Alice.

1. Measure, in inches, the height of several girls in your class who volunteer to be measured. Calculate the average height of your sample.

2. Assume that before she drank from the bottle, Alice was the average height you found in problem 1. How many times as tall was she after shrinking? (Is the scaling factor more than 1, or less than 1?)

3. a. Measure a real pencil or pen.

   b. Calculate the correct size for a pencil or pen of the same kind for Alice. Write an explanation of how you arrived at this measurement.

   c. Draw an actual-sized picture of a pencil for Alice based on your calculation in part (b).

4. Measure a real door, and calculate what the dimensions of "the little door into that lovely garden" might be.

> "Curiouser and curiouser!" cried Alice (she was so much surprised, that for the moment she quite forgot how to speak good English). "Now I'm opening out like the largest telescope that ever was! Goodbye, feet!"...
>
> … Just at this moment. her head struck against the roof of the hall: in fact she was now rather more than nine feet high...

5. Now, Alice is how many times as tall as the average height you found in problem 1?

6. What would be the size of a pencil if it were the right size for giant Alice? Show your calculations.

Scaling Projects

Creative Project: Your Own Story

Imagine that you are writing and illustrating a story for a young child featuring miniature people or giants. List and draw some of the objects and characters in the story, and give their dimensions. Make sure all of the dimensions are scaled correctly.

On a separate piece of paper, explain your calculations.

Creative Project: Your Own Math Problems

Ask a librarian or an elementary school teacher to suggest a book that involves miniature people or giants. Make up math problems based on the book. Use specific quotations from the book as much as possible.

On a separate piece of paper, solve the problems you make up.

Research Project: The Big Friendly Giant

The following quotations describe the Big Friendly Giant, a character in Roald Dahl's book *The BFG*.

a. "It was four times as tall as the tallest human."
b. "It actually had to bend down to peer into the upstairs windows. That's how tall it was."
c. "...an arm as thick as a tree trunk..."
d. "The Giant was sprinting down the High Street... Each stride he took was as long as a tennis court."
e. "In the middle of the floor there was a table twelve feet high..."
f. "He had truly enormous ears. Each one was as big as the wheel of a truck..."

Estimate the height of the Giant using the information given in each quotation. Explain your work.

◊ What real-world numbers did you use?

◊ How did you find them?

◊ What calculations did you do?

◊ Did the results of your calculations agree with each other?

◊ Based on all the calculations, what is your final estimate of the Giant's height?