Infinity: An Alternate Elective After Algebra 2

Henri Picciotto
The Urban School of San Francisco

math-ed@picciotto.org
www.picciotto.org/math-ed

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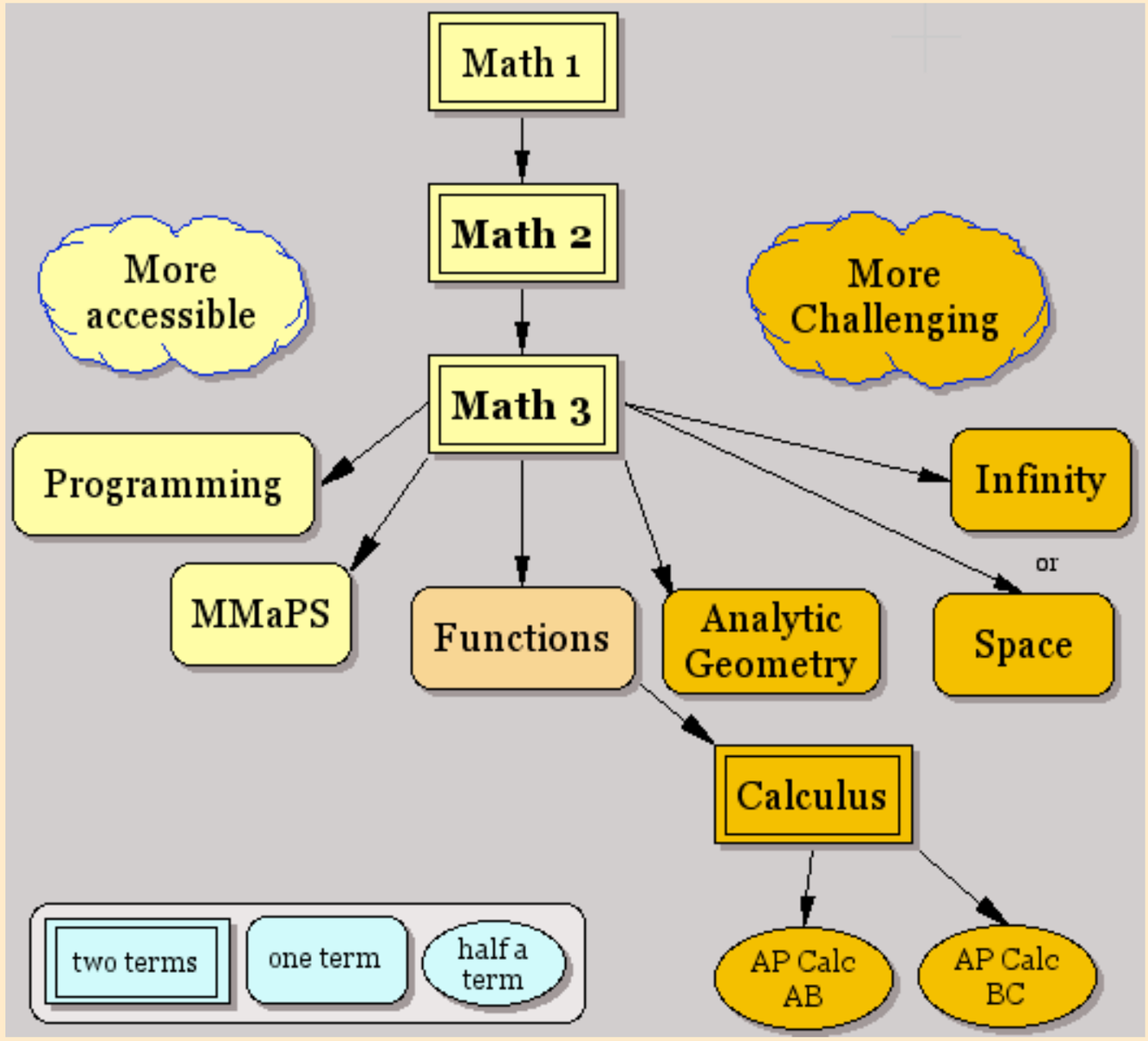
Math Courses



Infinity overview

Who takes the class

Juniors, before Calculus

Seniors, instead of or in addition to Calculus

Four topics

Readings

Algebra review

Computer tools

Infinity overview

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Infinite sets

Proof

Chaos

Fractals

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Galileo,
Jorge Luis Borges,
Douglas Hofstadter,
Martin Gardner,
Lewis Carroll,
James Gleick,
Scientific American,
...

Infinity overview

Who takes the class

Four topics

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Algebra review

Computer tools

prime numbers,
algebraic fractions,
similarity,
proportions,
sequences and series,
iteration,
logarithms,
complex numbers,
...

Infinity overview

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Fathom

Boxer

Galileo

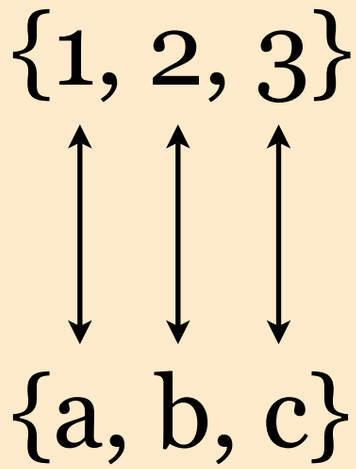
Consider the set S = {1, 2, …, n }, where n is a natural number. What fraction of the elements of S are perfect squares? (Hint: first figure it out for n = 1, n = 2, and so on, making a table.)

Georg Cantor

1845-1918

Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:



Equivalence

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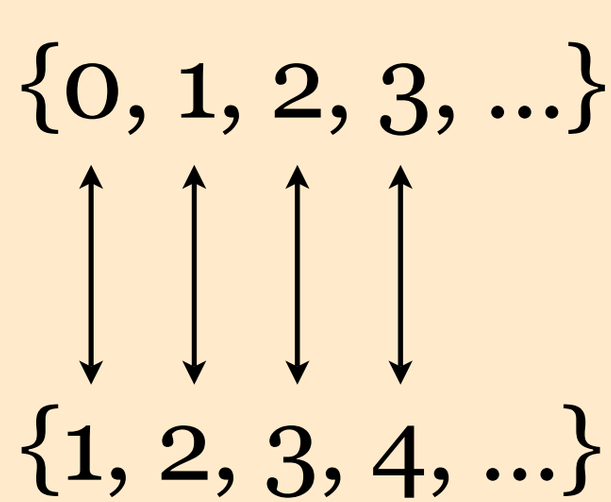


Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:

[5, 7] and [12, 19]

Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:

$[0,\infty)$ and $(0,\infty)$

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is a natural number} \\ x & \text{otherwise} \end{cases}$$


"HILBERT'S HOTEL" ©2002 CAROL LAY
THE INFINITE HOTEL WAS ALWAYS FILLED TO CAPACITY.
HILBERT'S HOTEL ∞
No Vacancy
WWW.WAYLAY.COM
YET, IF A NEW GUEST ARRIVED, SHE WAS ALWAYS GIVEN A ROOM.
AFTER ALL, THERE WERE AN INFINITE NUMBER OF ROOMS.
ROOM 843729486,714392645, 913653898652,732 NEEDS FRESH TOWELS.

Thinking about Infinity

Prime Numbers

Proof by contradiction

Countable Infinite Sets

An infinite set is said to be *countable* if it is equivalent to the natural numbers.

Example:

the integers

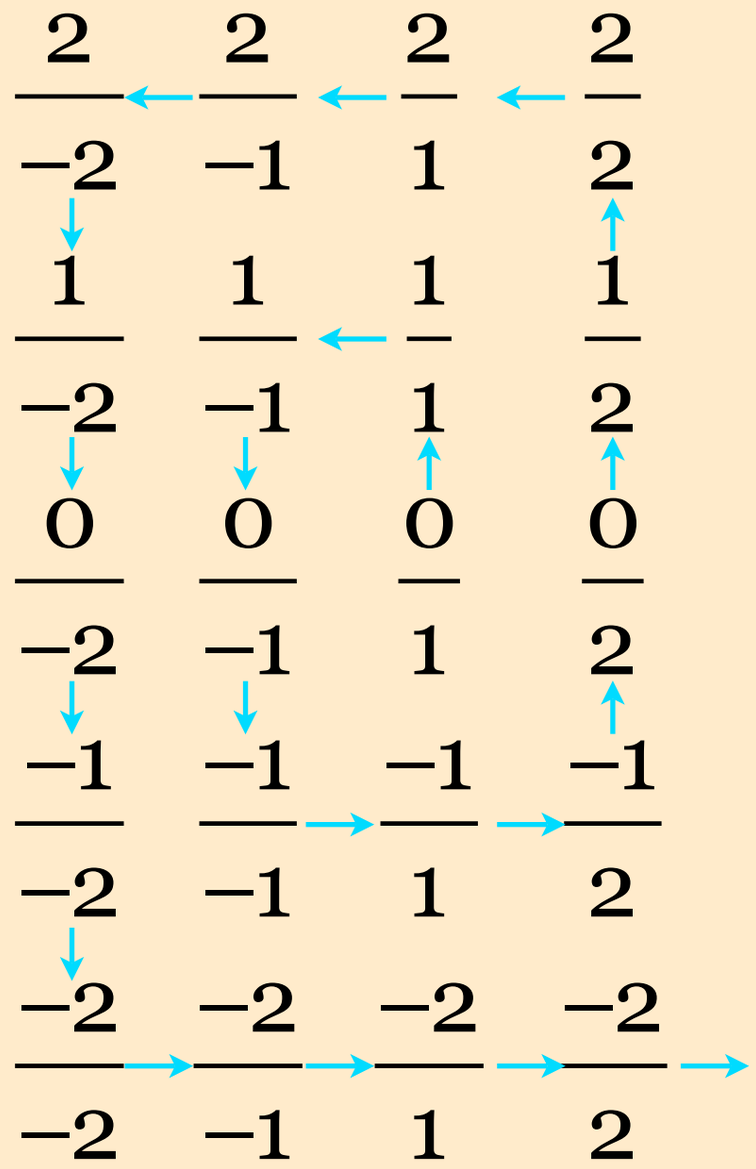
{0, 1, -1, 2, -2, 3, -3, ...}

Countable Infinite Sets

An infinite set is said to be *countable* if it is equivalent to the natural numbers.

Example:

the rationals



"The Power of the Continuum"

The set of real numbers in the interval [0, 1] is not countable

$$
\begin{aligned}
r_1 &= 0.\,d_{11}\;d_{12}\;d_{13}\;d_{14}\;d_{15}\;\cdots \\
r_2 &= 0.\,d_{21}\;d_{22}\;d_{23}\;d_{24}\;d_{25}\;\cdots \\
r_3 &= 0.\,d_{31}\;d_{32}\;d_{33}\;d_{34}\;d_{35}\;\cdots \\
r_4 &= 0.\,d_{41}\;d_{42}\;d_{43}\;d_{44}\;d_{45}\;\cdots \\
r_5 &= 0.\,d_{51}\;d_{52}\;d_{53}\;d_{54}\;d_{55}\;\cdots \\
&\vdots \\
r &= 0.\,d_1\;d_2\;d_3\;d_4\;d_5\;\cdots
\end{aligned}
$$

The Devil's Challenge

Raymond Smullyan

Infinite sets

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The Strong Law of Small Numbers

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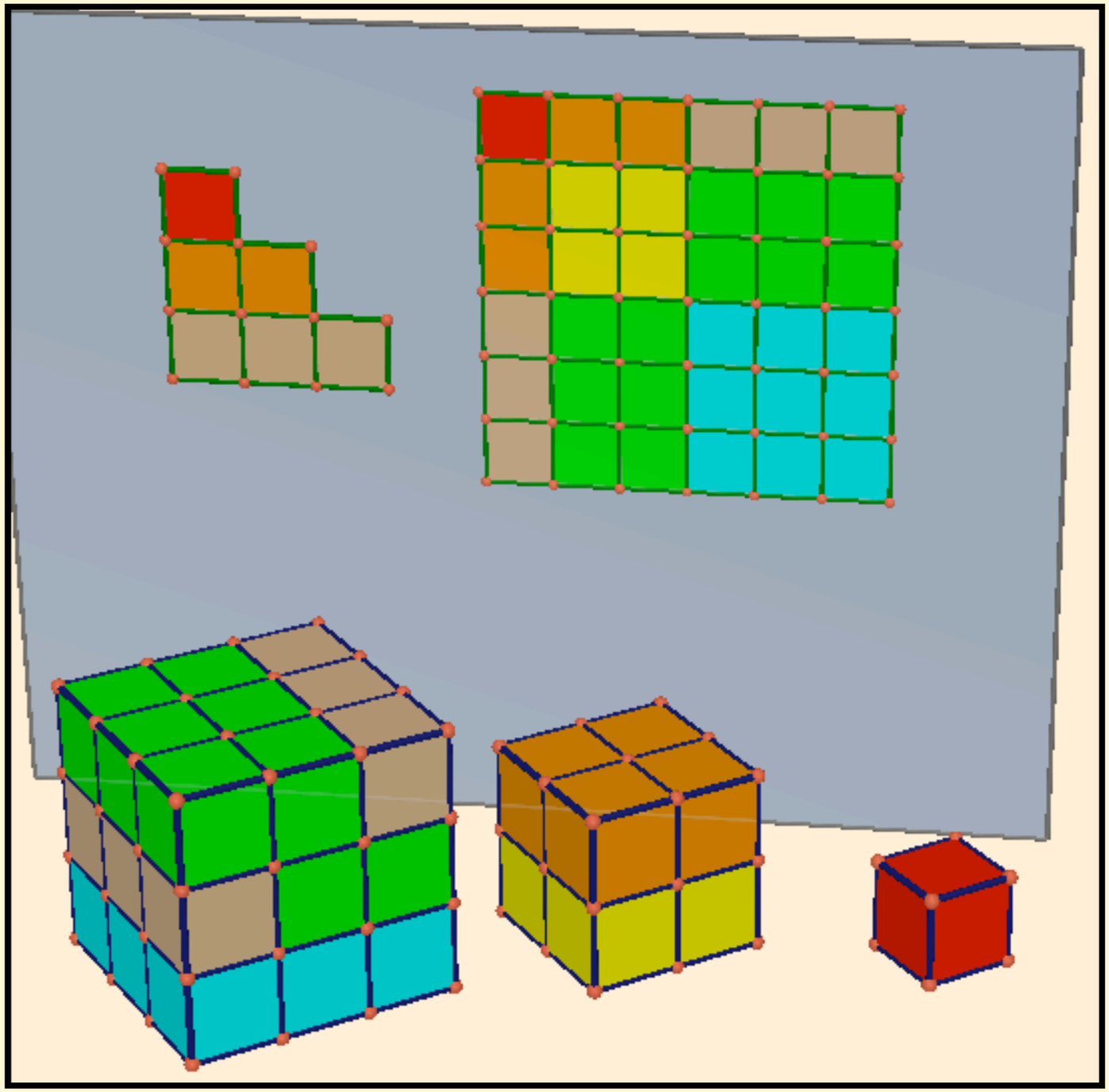
It seems like a power of 4 minus 1 is always a multiple of 3

=	n caseIndex − 1	k $4^n - 1$	kover3 $\frac{k}{3}$
1	0	0	0
2	1	3	1
3	2	15	5
4	3	63	21
5	4	255	85
6	5	1023	341
7	6	4095	1365
8	7	16383	5461
9	8	65535	21845

Will this pattern break down?

Generating conjectures



Fibonacci conjectures

Fibo

	n	F	L	test
=	caseIndex	if(n > 2){	switch( )	F · L
1	1	1	1	1
2	2	1	3	3
3	3	2	4	8
4	4	3	7	21
5	5	5	11	55
6	6	8	18	144
7	7	13	29	377
8	8	21	47	987
9	9	34	76	2584
10	10	55	123	6765

An explicit formula for Fibonacci numbers?

$$\left\lceil e^{\frac{n-1}{2}} \right\rceil$$

Iterating Functions in Fathom

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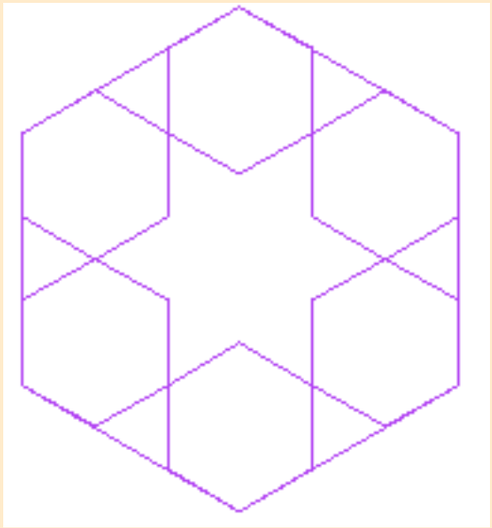
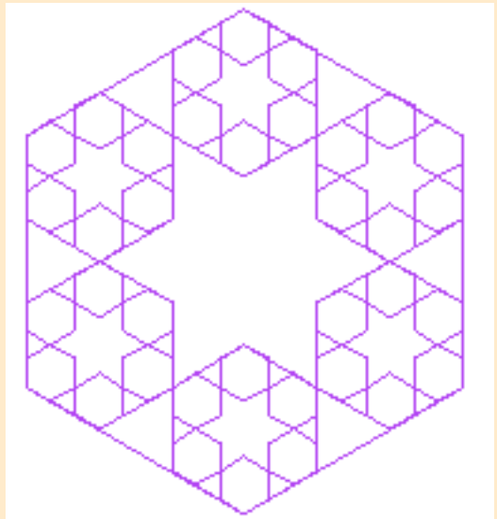
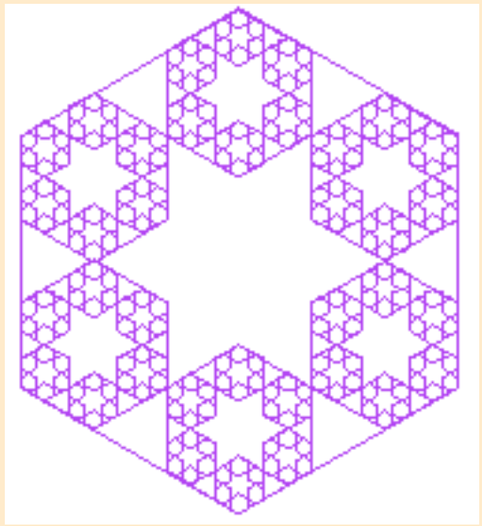
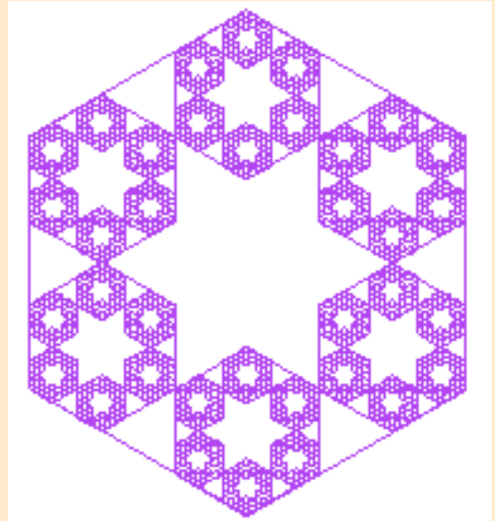

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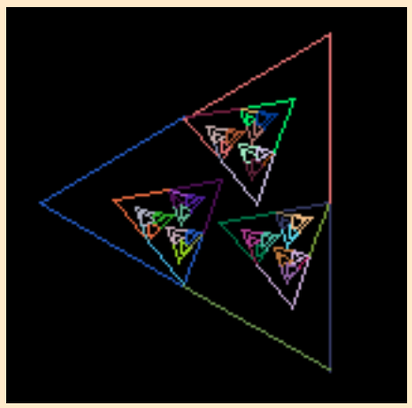

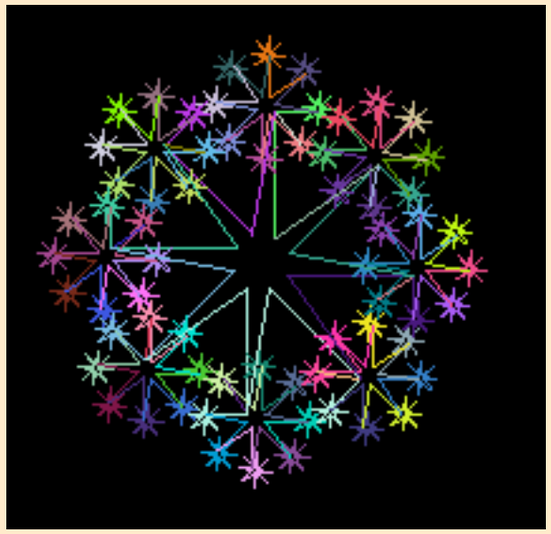

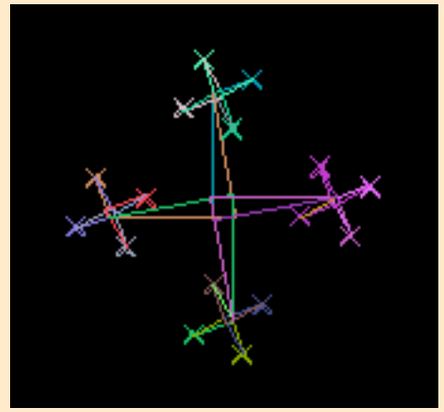

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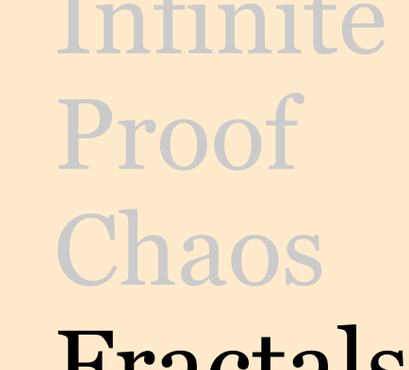

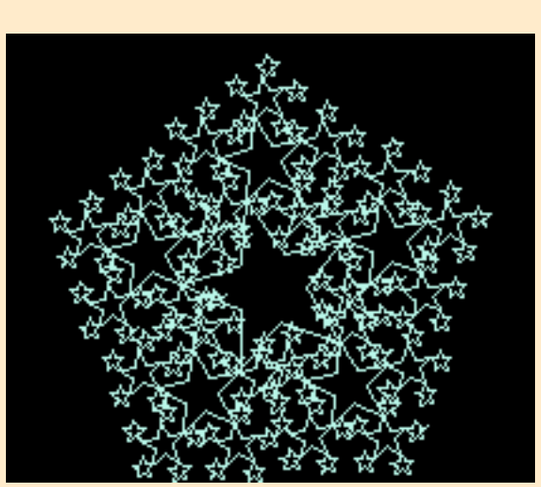

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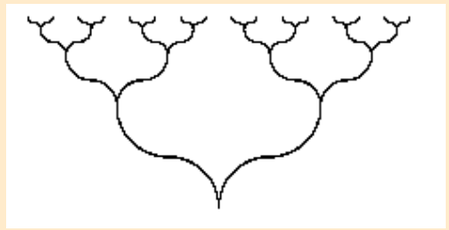

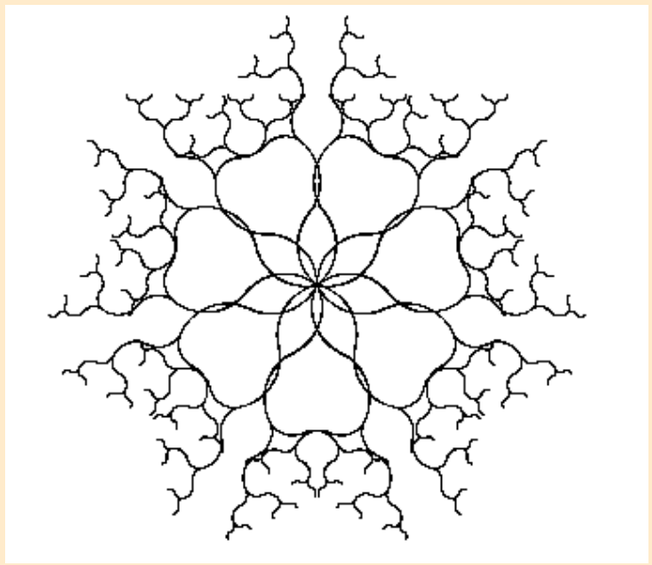

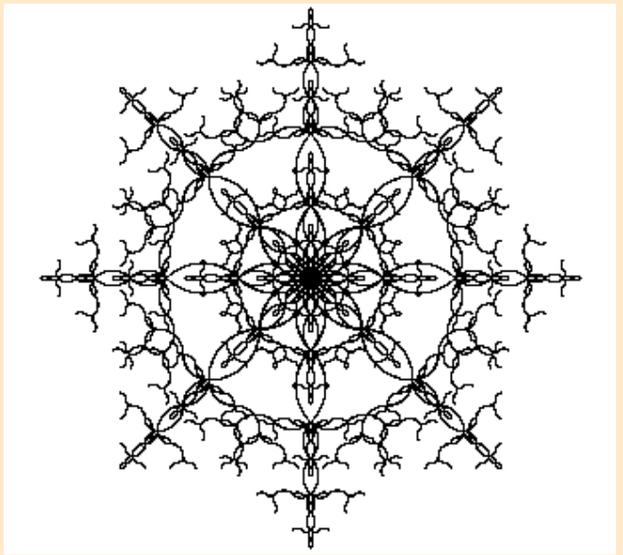

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