Using Graph Paper to Multiply and Divide Fractions

by Henri Picciotto

This is an approach based on graph paper, or dot paper, or geoboard.

**Multiplication**

First, we extend the area model of multiplication to fractions. The key is to choose a convenient rectangle to represent 1. Let’s say I want to multiply \( \frac{2}{3} \) by \( \frac{1}{4} \). For this problem, a convenient rectangle for 1 would be 3 by 4, because it would be easy to represent \( \frac{2}{3} \) and \( \frac{1}{4} \) on it, as in the figure. Since for a rectangle, length times width equals area, we need to figure out which fraction would represent the rectangle with dimensions \( \frac{2}{3} \) and \( \frac{1}{4} \). It consists of two squares out of the twelve that make 1. So we conclude:

\[
\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12}
\]

This basic approach will still work with fractions that are greater than 1. To multiply \( \frac{4}{3} \) by \( \frac{1}{4} \), I still use a 3 by 4 rectangle for 1. and find that:

\[
\frac{4}{3} \cdot \frac{1}{4} = \frac{4}{12}
\]

In both cases, simplifying the answer is easy to do by comparing the resulting rectangle to 1.

**Division**

Let us first see what happens when using the area model to divide by smaller and smaller numbers. We will arrange the divisions in a manner similar to the layout for long division. For example, 6 divided by 2 equals 3, or \( \frac{3}{2} \) would look like this, with 6 as the area of the rectangle, the divisor 2 as the height, and the quotient 3 as the width:
These rectangles represent 6 divided by 6, 3, 2, and 1:

As the divisor gets smaller, the quotient gets bigger. (In fact, we see that when we divide the divisor by n, we multiply the quotient by n.) Continuing that process, we represent the division of 6 by 1, ½, and ⅓:

The quotient continues to get bigger, and in fact in order for the area to still be 6, we see that when we divide 6 by 1/n, the quotient must be 6 times n. This makes sense, as it continues the pattern we saw above.

We conclude that to divide by a unit fraction, multiply by its reciprocal.

But what if we are dividing by, say, ⅓? Since we are multiplying the divisor by 2 (as compared to 6 divided by ⅓), we have to divide the quotient by 2:

So to divide 6 by m/n, we multiplied by n, and divided by m, which amounts to multiplying by n/m.

In short: to divide by a fraction, multiply by its reciprocal.