CONSTRUCTION

Bisector Theorems

Make two conjectures:

1. The set of points that are equidistant from the endpoints of a line segment is the ____________________ ____________________ of the line segment.

2. By definition, the distance from a point to a line (or ray, segment) is always the __________________ distance!

3. The set of points that are equidistant from the rays of an angle is the ______________ _________________.

4. Write each of the conjectures in if-then form, accompanied by an if-then diagram.

5. Write the converse of each conjecture, accompanied by an if-then diagram.

6. Prove four theorems.

Basic Constructions

Construct the following with compass and straightedge:
(the spaces on this sheet are for sketches,
do the constructions on other paper)

1. All points that are equidistant from a given point

2. All points that are equidistant from the endpoints of a segment

3. All points that are equidistant from the sides of an angle

4. The distance from a point to a line

5. An equilateral triangle

6. An isosceles triangle that is not equilateral

7. A rhombus

8. A square

9. Bonus: Some points equidistant from a point and a line

Cabri Construction Challenges

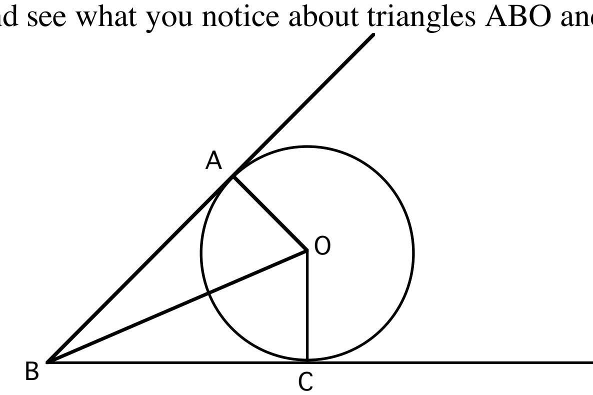
For each problem, start by drawing the given in Cabri, then do the construction. *The construction is not correct if you can ruin it by moving the given.*
At each checkpoint (√), show your teacher your figure, and save it. You can call the files **cp3**, **cp4**, **tc3**, etc, or use names that are more meaningful.

Circles Through Points

1. Given a segment, construct a circle that has the segment as its diameter.
2. Given two points, construct three different circles that go through both. **Hint**: first find where the centers should be located.
3. Given three points, construct a circle that goes through all three. **Hint**: use what you found out in problem 2. √
4. Given a rectangle, construct a circle through its vertices. √

Tangent Circles

1. Given a line and a point not on it, construct a circle centered at the point and tangent to the line.
2. Given two lines, construct three different circles tangent to both. **Hint**: first look at this figure, and see what you notice about triangles ABO and CBO.



3. Given a triangle, construct a circle tangent to all three sides. **Hint**: use what you learned in problem 2. √
4. Given a triangle, construct a circle tangent to one side, and to the extensions of the other two, on the outside of the triangle. √
   (Optional: given a triangle, construct four circles tangents to the three sides — one inside, and one outside on each side.) √
5. Given a line and a point not on it, construct a circle tangent to the line, which passes through the point.
6. Find three more circles that satisfy the conditions of problem 5. √
7. Given a circle and a point not on it, construct a circle through the point, tangent to the circle.
8. Find three more circles that satisfy the conditions of problem 7. √

Constructing Tangent Lines

Important: All these constructions depend on knowing that a line tangent to a circle is ____________ to the ________ at the point of contact.

1. Given a circle and a point on it, construct a tangent to the circle through the point.

2. Given a circle and a line, construct a tangent to the circle that is parallel to the line.

3. Given a circle and a line, construct a tangent to the circle that is perpendicular to the line.

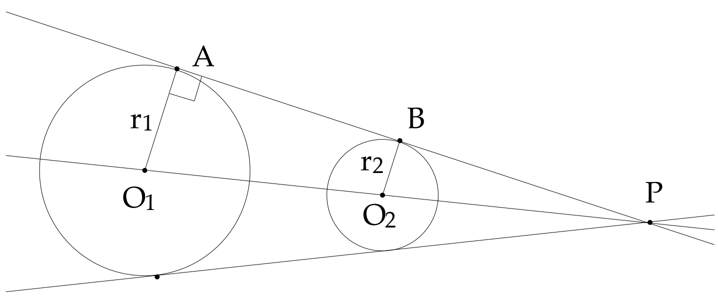
Before the next tangent line construction, you need to solve this problem:

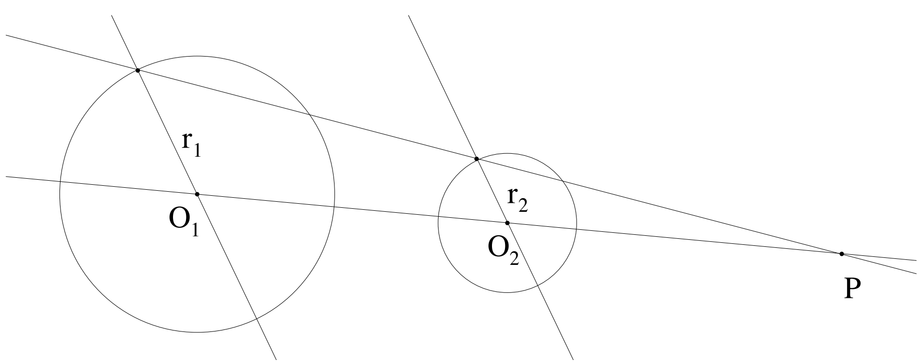
4. Given a line segment, construct three right triangles that have the segment as their hypotenuse. (Hint: use something you know about inscribed angles.)

5. Complete the sentence, using what you learned in #4: Given two points A and B, the location of all the points P such that $\angle$ APB = 90° is __________________________

6. Given a circle and a point outside it, draw both tangents to the circle through the point.

Challenge! The next two constructions are tricky. Study these figures to help you with #7. (Hint: Think of similar triangles, and show that $PO_1 / PO_2 = r_1 / r_2$ in both figures.)





7. Given two non-congruent circles, draw a line tangent to both, on the outside.

8. Given two non-congruent disjoint circles, draw a line tangent to both, on the inside.

Lines in a Triangle

Patty Paper

Theorem: The perpendicular bisectors of the sides of a triangle meet in one point, the center of the circumscribed circle.
(The *circumscribed circle* is the circle through the vertices of a triangle.)

1. Proof: write the proof of the theorem, based on your teacher's explanations.

2. Using a straightedge, draw a large triangle on patty paper. Label the vertices A, B, C. Make the perpendicular bisectors of all three sides by folding. Call their meeting point O.

3. Using a compass, draw the circumscribed circle.

Theorem: The angle bisectors of the angles of a triangle meet in one point, the center of the inscribed circle.
(The *inscribed circle* is the circle tangent to the three sides of a triangle.)

4. Proof: write the proof of the theorem. It is not unlike the proof above.

5. Using a straightedge, draw a large triangle on patty paper. Label the vertices A, B, C. Make the angle bisectors of all three angles by folding. Call their meeting point P.

6. Drop a perpendicular by folding, from P to one of the sides of the triangle.

7. Using a compass, draw the inscribed circle.

Cabri

Definition: In a triangle, the line connecting a vertex to the midpoint of the opposite side is called a *median.*

8. Using Cabri, draw a triangle ABC. Let D and E be the midpoints of AC and BC respectively. Draw the medians AE and BD. Call the point where they meet M. Let F and G be the midpoints of BM and AM respectively.

9. Draw the quadrilateral DEFG.
   a. Prove it is a parallelogram. (Hint: how is DE related to AB? How is GF related to AB?)
   b. Prove that M is 1/3 of the way down the median from the side. (Hint: the diagonals of a parallelogram bisect each other.)
   c. Prove that the third median must pass through M. (Hint: use what you proved in #9b.)

Theorem: The medians of a triangle meet in a single point, called the *centroid* or center of gravity of the triangle.

Definition: In a triangle, the perpendicular to a side through the opposite vertex is called the *altitude.*

10. Using Cabri, draw a triangle ABC. Construct parallels to the three sides, through the opposite vertices, creating a new, larger triangle DEF.

11. Prove that the altitudes of Δ ABC meet in a single point. Hint: the altitudes of ΔABC are what in ΔDEF?

Theorem: The altitudes of a triangle meet in a single point, called the *orthocenter*.

The following result is difficult to prove, but is fun to discover.

12. Using Cabri, draw a triangle ABC. Construct all four centers, but hide all the construction lines. Which three centers are *always* collinear?