Construction: Teachers' Notes

Introduction

At the Urban School of San Francisco, we teach this unit during the second half of Math 2, our geometry course.

Like proof, construction is about logic: given a starting configuration of points and lines, how does one correctly create a target figure if one is only allowed a given set of tools? There is a puzzle-like quality to these questions, and tough construction challenges are difficult even for the best students. Like many appropriately chosen puzzles in the classroom, they can be very motivating.

There is an ancient tradition of compass and straightedge construction, of course, and our approach definitely fits in that history. However we have found that we get more educational mileage by diversifying the toolbox. We still use the compass to make circles, to copy line segments, and to bisect segments and angles, but we use patty paper (tracing paper) to copy angles. We also use *interactive geometry*: computer-aided construction using Cabri to pursue a range of construction challenges. There are of course other interactive geometry software applications, and these lessons can be done with any of them — Geometer's Sketchpad, Cinderella, etc.

There are a number of important results in the unit: the perpendicular bisector of a segment, the angle bisector, the various centers of triangles and their properties, tangents to circles, inscribed and circumscribed circles, and so on.

This work can lead to a geometric introduction to the conic sections.

If Soccer Angles (*Geometry Labs,* Lab 1.10) was done previously, it can be followed up towards the end of this unit with a construction of the point with the optimal shooting angle.

Caution: it is imperative you work through the sheets yourself before introducing them to your class.

Bisector Theorems

It is very effective to precede this lesson with a quick outside activity. We usually do it in the gym, but any large space will do.

Ask students to arrange themselves so that they are:

0. Equidistant from a point (a given student)
1. Equidistant from the endpoints of a segment (two students)
2. Equidistant from the rays of an angle (for example two intersecting lines on the basketball court, or two consecutive edges of a lawn, etc.)

Each time, have the students discuss where they had to place themselves.

Basic Constructions

For the first few problems, just tell the students how to do it. (It can be found in just about any geometry textbook.) For the remaining problems, give generous hints.

Hints:

7, 8. Start with the diagonals. (See *Geometry Labs*, Lab 6.3: Making Quadrilaterals from the Inside Out)

9. Here is a solution students are not likely to find: draw any perpendicular L to the original line D. Perpendicularly bisect the segment connecting the intersection of L and D with the original point F. The intersection of the perpendicular bisector and L has the required property. Doing this many times, one gets points on a parabola.

Cabri Construction Challenges

This page assumes students have already been introduced to Cabri. There is more than one period's work here!

Encourage students to help each other. Be stingy with the hints until #6-8. For #6, see the hint for #9 of **Basic Constructions**. For #8, follow the same logic, starting with a diameter of the circle for line L. For other, simpler constructions, see <www.MathEducationPage.org/conics>.

Constructing Tangent Lines

Prerequisite: the theorems about tangents to a circle.

Lines in a Triangle

This is where we introduce the (important!) concurrence theorems. There is more than one period's work here!

Prerequisites: For #1-7, the theorems about bisectors, and their converses. For #8-12: the midsegment theorem.

You will need to prove #1, as it is almost certain that students will not be able to do this on their own.

After #9, you can ask students to find the centroid of a cardboard triangle, and have them balance the triangle on a finger. It is not necessary to do a formal construction: just use a ruler to find the midpoints.