

# Complicated Areas

You will need:

geoboards

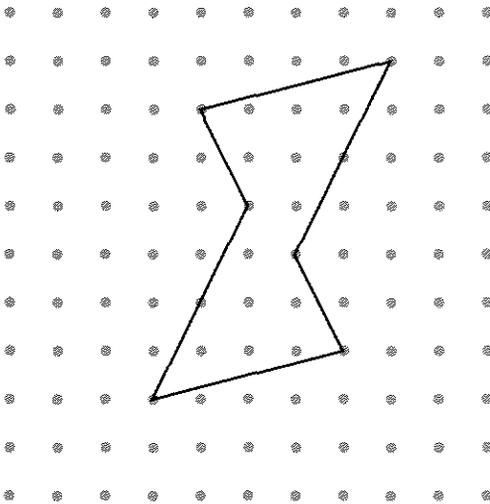


dot paper



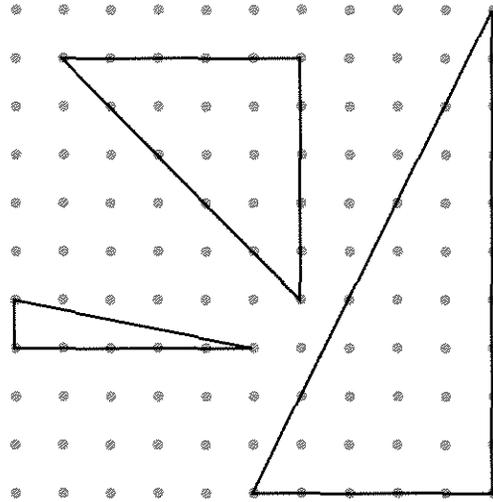
## DISCOVERING AN AREA FORMULA

1. a. Find the area of this figure.
- b. Explain how you did it, with the help of illustrations on dot paper.
- c. Compare your approach with other students' work.



In the figure above, the rubber band is in contact with 8 geoboard pegs (which we will call *boundary dots*). The figure encloses 12 inside pegs, which we will call *inside dots*.

2. For each figure, give the number of boundary dots, the number of inside dots, and the area.



3. **Exploration** Try to figure out the relationship between boundary dots, inside dots, and area. (Hints: Sketch many simple figures, count their dots, and find their areas. Keep detailed and clear records. Start by working on the problem for figures having zero inside dots, then one inside dot, and so on.) Keep records of your work in a table like this one.

Boundary Dots	Inside Dots	Area
...	...	...

4. Make three figures having 3 boundary dots and 0 inside dots. Find the area of each figure.

5. Make three figures having 4 boundary dots and 0 inside dots. Find the area of each figure.
6. Make three figures having 5 boundary dots and 0 inside dots. Find the area of each figure.
7. 
  - If two figures have no inside dots and the same number of boundary dots, what can you say about their areas?
  - What happens to the area if the number of boundary dots increases by 1?
8.
  - Predict the area of a figure having 10 boundary dots and 0 inside dots.
  - Check your prediction by making three such shapes and finding their areas.
  - What would the area of a figure having 99 boundary dots and 0 inside dots be?
9. **Generalization** Explain how one could find the area of a figure having  $b$  boundary dots and 0 inside dots, without making or drawing the figure.
10. Make figures having 10 boundary dots and 1, 2, 3, etc. inside dots. For each one, find its area. Keep your work organized in a table.
11.  What happens to the area when the number of inside dots increases by 1?
12.
  - Predict the area of a figure having 10 boundary dots and 10 inside dots.
  - Check your prediction by making three such shapes and finding their areas.
  - What would the area of a figure having 99 boundary dots and 101 inside dots be?
13. **Generalization** Explain how one could find the area of a shape having  $b$  boundary dots and  $i$  inside dots, without making or drawing the figure. You have discovered *Pick's Formula*.
14. Use the result from problem 13 to check your answers to problems 1 and 2.


**DISCOVERY** PATTERNS AND FUNCTIONS

15. Find as many functions of  $x$  as possible whose value is 5 when  $x$  is 2.
16. Multiply.

$$\left(1 - \frac{1}{7}\right) \cdot \left(1 - \frac{2}{7}\right) \cdot \left(1 - \frac{3}{7}\right) \cdot \dots \cdot \left(1 - \frac{9}{7}\right)$$

17. 1

$$1+2+1$$

$$1+2+3+2+1$$

$$1+2+3+4+3+2+1$$

What do you notice about these sums? Explain the pattern, using a figure if you can.