

Polynomial Functions

You will need:

graph paper



Definition: A *polynomial function* is a function of the form $y = a$ polynomial.

1. **Exploration** Which of these polynomial functions do you think have graphs that are straight lines? Which have curved graphs? Explain why you think so.

a. $y = x^2$ b. $y = 2x - 1$
 c. $y = 2x^2$ d. $y = x^3$

ORDER OF OPERATIONS

2. Make a table of at least eight (x, y) pairs for each function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table. Label each graph with its equation. You will need to refer to these graphs later.

a. $y = x^2$ b. $y = x^3$

To make a table of values for graphing $y = -x^2$, we have to know what the expression $-x^2$ means. Does it mean *square x , then take its opposite* or *take the opposite of x , then square it*? Which operation should be done first?

To avoid this kind of confusion, mathematicians have agreed on the following rule.

Rule: *Exponentiation should be performed before other operations.* To change this order, we have to use parentheses.

Examples:

- $-x^2$ means *square x , then take the opposite.*
- $(-x)^2$ means *take the opposite of x , then square the result.*

3. Make a table of at least eight (x, y) pairs for each function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table. Label each graph with its equation.

a. $y = (-x)^2$ b. $y = -x^2$

4. Compare your graphs in problem 3 with the graph of $y = x^2$. Explain what you observe.

5. Graph these polynomial functions.

a. $y = -x^3$ b. $y = (-x)^3$

6. Compare your graphs in problem 5 with the graph of $y = x^3$. Explain what you observe.

DEGREE

Definition: The degree of a polynomial function in one variable is the highest power of the variable that appears in the polynomial.





Examples: $y = x^3$ and $y = x^2 + 2x^3$ are both third-degree polynomial functions. The equation $y = 2x$ is first-degree, and the equation $y = 1$ is zero-degree.

7. What is the degree of each of these polynomial functions?

a. $y = 5 + x^2 - x$
 b. $y = 4x^3 - 3x^2 + 5$
 c. $y = 45$

8. Make a table of at least eight values for each third-degree function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table.

a. $y = 2x^3$ b. $y = x^3 + 1$
 c. $y = -x^3 - 2$

9. Repeat problem 8 for these second-degree functions.
 a. $y = x^2 - 1$ b. $y = -3x^2$
 c. $y = -x^2 + 2$
10. Graph these first-degree functions.
 a. $y = 5x$ b. $y = x$
 c. $y = -2x + 1$
11. Graph these zero-degree functions.
 a. $y = 4$ b. $y = -3$
 c. $y = 0$
- THE EFFECT OF DEGREE**
12.  Tell whether each sentence (a-b) could describe the graph of a zero-degree, first-degree, second-degree, or third-degree polynomial function. More than one answer may be possible for each description.
 a. The graph is a straight line.
 b. The graph is a curve.
13.  Repeat problem 12 for these descriptions.
 a. The graph goes through the origin.
 b. The graph never crosses the x -axis.
 c. The graph never crosses the y -axis.
14.  Repeat problem 12 for these descriptions.
 a. The graph passes through quadrants I and III only.
 b. The graph passes through quadrants II and IV only.
 c. The graph passes through quadrants I and II only.
15. **Summary** How does the degree of the equation affect its graph? Write a summary explaining everything you know about this.
16. 
 a. Make a table of values and graph the function $y = 24/x$.
 b. Is this a polynomial function? Explain.

REVIEW POSSIBLE OR IMPOSSIBLE?

17. Decide whether each of the following situations is possible or impossible. If it is possible, give an example. If it is impossible, explain *why* it is impossible. Can you subtract
- a negative number from a negative number to get a positive number?
 - a negative number from a negative number to get a negative number?
 - a negative number from a positive number to get a positive number?
 - a negative number from a positive number to get a negative number?
 - a positive number from a negative number to get a negative number?
 - a positive number from a negative number to get a positive number?