

# Essential Ideas



## WORKING BACKWARDS

Abe and Bea had baked a batch of cookies. They told Reg, Al, and Lara that they could each have one-third of the cookies. Later, Reg went into the kitchen and took one-third of the cookies. An hour after that, not knowing that Reg had already taken his share, Lara claimed one-third of the remaining cookies. A few minutes later Al, thinking he was the first to find the cookies, devoured one-third of what was left.

- If 8 cookies are left, how many must Abe and Bea have baked?

## TWO NEGATIVES

- Find the sign of the result.
  - $3 - 5$
  - $3 - (-5)$
  - $-5 - (3)$
  - $-5 - (-3)$
- Find the sign of the result.
  - $-(-5)(-3)$
  - $-(5 - 3)$
  - $-[-3 - (-5)]$
  - $-(-5)(-3)$

## POSITIVE, NEGATIVE, OR ZERO?

- For each expression, write *P*, *N*, and/or 0, depending on whether it can possibly be positive, negative, or 0. (Try various values for the variables to help you decide. For example, -2, 0, and 2.) Explain your answers.
  - $5x$
  - $-2x^2$
  - $-9y$
  - $5y^2$
  - $z^3$
  - $-a^4$

## SIMPLIFYING EXPRESSIONS

Simplify each expression.

- $12x - 6xy - (-3x) - (-2y)$
- $-3x^2 - (3)2 + x^2 - (2 - x^2)$
- $x - (x - 5) - (5 - x)$

## FROM WORDS TO ALGEBRA

- Translate each step into algebra.
  - Think of a number.
  - Add 4.
  - Multiply the result by 2.
- If I got 46, what was my original number?
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  - Multiply by 2.
  - Add 4.
- If I got 46, what was my original number?
- Compare your answer to part (b) with your answer to part (b) in problem 8. Were your answers the same or different? Explain.

## COMPARING EXPRESSIONS

- Find a value of  $x$  for which
  - $-8x - 1$  is less than  $8x + 3$ ;
  - $-8x - 1$  is greater than  $8x + 3$ ;
  - $-8x - 1$  is equal to  $8x + 3$ .

## MULTIPLICATION TABLES

Find these products. Combine like terms.

- $(x + 3)(2x + 4)$
- $(x + 3)(2x + 4y)$
- $(x + 3 + y)(2x + 4y)$

Fill in the blanks.

14. 

$x$	$-3$	$5y$
$2x^2$	$-6x$	$10xy$

15. 

_____	_____	_____
$3y$	$-6x^2y$	$15y^3$
	$-3y$	

16. \_\_\_\_\_  $(x - 2) = 2 - x$

### OPPOSITES AND RECIPROCAL

17. Simplify each expression. Look for shortcuts.

a.  $9 \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot 5 \cdot \frac{3}{2}$

b.  $[5x - (-5x)] - [5x - (-5x)] - 16x$

c.  $0.5 \cdot 25 \cdot 0.02 \cdot 2$

18. Gabe and Abe were arguing about  $xy$ . Gabe said that the opposite of  $xy$  is  $yx$ . Abe said that the opposite of  $xy$  is  $-xy$ . Lara overheard them, and said she thought that the opposite of  $xy$  is  $-yx$ . Write an explanation that will settle their argument.

19. What numbers are

- greater than their reciprocal?
- less than their reciprocal?
- equal to their reciprocal?
- less than their opposite?
- equal to their opposite?



20. a. Which of the following is the reciprocal of  $3x$ ?

$$\frac{1}{3x}, \frac{3}{x}, \text{ or } \frac{1}{3}$$

- b. Check your answer by substituting two different numbers for  $x$  and showing that the product of  $3x$  and its reciprocal is 1 in both cases.

### INVERSE FUNCTIONS

Write the inverse of each of the following functions.

- The function adds 2 to  $x$  and multiplies the result by 4.
  - The function multiplies  $x$  by 4 and adds 2 to the result.
  -   $y = 7x - 4$
- The function takes the opposite of  $x$ .
  - The function takes the opposite of  $x$ , adds 5, and divides the result by 2.
  -   $y = \frac{3-x}{6}$

Scientists sometimes use the Kelvin temperature scale. To convert Kelvin temperatures to Celsius, you subtract 273. For example, the melting temperature of iron is 1808° Kelvin, or 1535° Celsius.

- Lead melts at 600° Kelvin. What temperature is that in Fahrenheit? (Use the information from Lesson 8.)
- Explain how to convert Kelvin temperatures to Fahrenheit, and how to convert Fahrenheit to Kelvin. (Hint: Use arrows to show each step of the conversion.)
- Make a function diagram for the function  $y_1 = (x/2) + 1$ .
  - Make the function diagram of its inverse and find the rule.
  - Find the function that results from combining  $y_1$  and its inverse. Does the order in which you combine the functions matter? Explain.

### SOLVING EQUATIONS

Use the cover-up method to solve these equations.

26.  $\frac{24}{x-5} + 3 = 9$     27.  $\frac{x-5}{24} + 3 = 9$

28.  $\frac{5-x}{24} + 3 = 9$     29.  $\frac{24}{5-x} + 3 = 9$

- Compare the solutions to each pair of equations. (Use related multiplication equations.)
  - $\frac{2}{M} = 6$  and  $\frac{6}{M} = 2$
  - $\frac{8}{M} = 4$  and  $\frac{4}{M} = 8$
  - $\frac{20}{M} = 5$  and  $\frac{5}{M} = 20$
  - Make up another example like this.
- Describe the pattern you found in problem 30. Explain why it works.