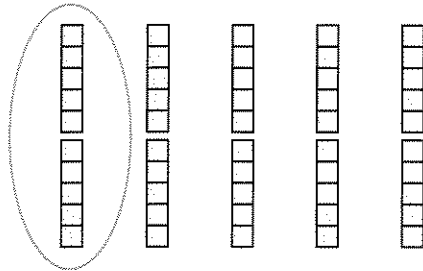


A MODEL FOR MULTIPLICATION

You cannot easily show multiplication by fractions with the Lab Gear, but the Lab Gear can help you think about it. For example, $(1/5) \cdot 50$ is read *one-fifth of fifty*. This means that we divide 50 into five parts and take one of them.

The diagram shows that $(1/5) \cdot 50 = 10$.



$(2/5)$ is two of five parts, so $2/5 \cdot 50 = 20$.

- Find a number you could multiply by 8 to get a number less than 8.
- Without finding its value, decide whether x would be more or less than 1. Explain how you know.
 - $8 \cdot x = 50$
 - $8 \cdot x = 5$
 - $8 \cdot x = 0.05$
- Find the value of x for each equation in problem 2. (Hint: Remember that for any multiplication, there are two related divisions. You may use a calculator.)

A MULTIPLICATION SHORTCUT

- Take 8, 3, and 2. They are three numbers whose product is 48. Another multiplication possibility is $6 \cdot 4 \cdot 2$. Find as many ways of writing 48 as a product of three different numbers as you can. Do not use 1 as a factor.

- Exploration** Do not use 1 as a factor.
 - Write 2 as a product of two different numbers.
 - Write 4 as a product of four different numbers.
 - Write 6 as a product of six different numbers.
 - Write 12 as a product of twelve different numbers.


Definition: The product of a number and its reciprocal is 1. Another way of saying this is, *the reciprocal of a number is the result dividing 1 by the number.*

Examples: $3 \cdot 1/3 = 1$
 $2/3 \cdot 3/2 = 1$
 $0.31 \cdot 100/31 = 1$

- Explain how the reciprocals of 3, 2, and 0.31 may have been found for the examples above. (No calculator was used.)

Guess the value of x , *without using your calculator*. If you think about reciprocals you will have to do very little arithmetic.

- $5 \cdot \frac{1}{5} \cdot x = 6$
 - $4 \cdot x \cdot 9 \cdot \frac{1}{4} = 45$
 - $x \cdot 8 \cdot 7 = 8$
 - $x \cdot 8 \cdot 3 = 3$
 - $\frac{2}{3} \cdot x \cdot 3 \cdot \frac{1}{2} = 15$
- $2 \cdot x \cdot 3 = 2$
 - $x \cdot 2 \cdot 2 \cdot 9 \cdot 3 = 6$
 - $\frac{1}{5} \cdot (5x) \cdot 3 = 1$
 - $\frac{1}{5} \cdot (5x) = \frac{3}{5}$
- Make up two more equations like problems 7 and 8 and solve them.

10.  Find two numbers a and b that will satisfy each equation. *Don't use your calculator.* Instead, think about reciprocals. Do not use 1 for a or b .

a. $a \cdot b \cdot 14 = 28$ b. $a \cdot b \cdot 28 = 14$
 c. $\frac{2}{3} \cdot a \cdot b = 10$ d. $a \cdot b \cdot 10 = \frac{2}{3}$


RECIPROCAL ON THE CALCULATOR

Most scientific calculators have a key for reciprocals: $\boxed{1/x}$, or $\boxed{x^{-1}}$. (On calculators that do not have such a key, you can divide 1 by a number to find the number's reciprocal.)

11. Find the reciprocal of:
 a. $1/23$; b. 0.456 ; c. 7.89 .

12. **Report** What is the result when you
 a. multiply a number by its reciprocal?
 b. divide a number by its reciprocal?

Be sure your results work for all numbers. Explain how you reached your conclusions.

13. $1/82 < 0.0123 < 1/81$. Explain.
 14. Find two consecutive whole numbers such that 0.00123 is between their reciprocals.
 15. Repeat problem 14 for 0.000123
 16.  Explain your method for solving problems 14 and 15.

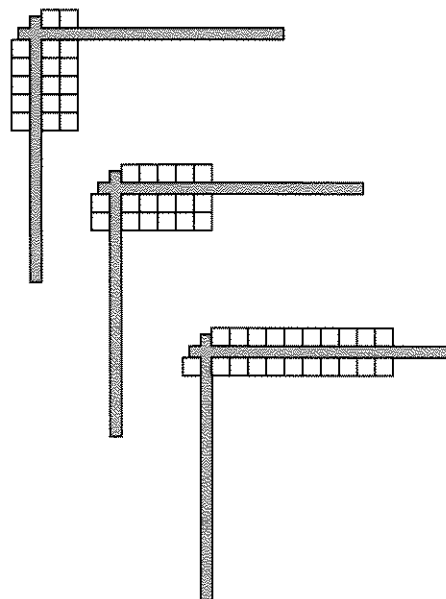
A MODEL FOR DIVISION

17. **Exploration** Find a positive number such that when you divide that number by 5, your answer is
 a. a number less than 1;
 b. a number between 10 and 20;
 c. a number greater than 100.

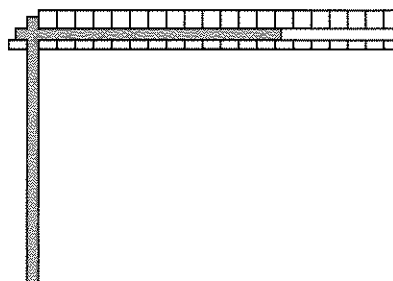
18. Find a positive number such that when you divide 5 by it, your answer is
 a. a number less than 1;
 b. a number between 10 and 20;
 c. a number greater than 100.


Division by numbers between 0 and 1 is hard to show with the Lab Gear.

These diagrams show $10/5$, $10/2$, and $10/1$.




What would $10/(1/2)$ look like? We cannot actually build this with the Lab Gear, but we could imagine what it would look like if we sliced each block in half.



19. a. What is the answer to the division shown in the figure?
 b. Dividing by $1/2$ is equivalent to multiplying by what number?
20. a. Will the result of the division $8/(1/4)$ be more or less than 8?
 b. Use a sketch to show the division $8/(1/4)$.
 c. What is the answer to the division?
 d. Dividing by $1/4$ is equivalent to multiplying by what number?
21.  a. What is the result of the division of 8 by 0.1, 0.01, 0.001?
 b. What would happen if you divided 8 by a number that is much smaller than 0.001, almost equal to zero?
 c. How about dividing 8 by 0?

A DIVISION SHORTCUT

22. a. If you multiplied 5 by a number and got 30, what was the number?
 b. If you divided 5 by a number and got 30, what was the number?
 c. Compare your answers to parts (a) and (b). How are these numbers related?

23.  *Dividing by a number is the same as multiplying by its reciprocal.* Explain, using examples.

Use this fact to perform each of the following divisions without your calculator.

24. $12/(1/4)$ 25. $12/(2/3)$
 26. $10/0.4$ 27. $x^2/(1/x)$

SMALL NUMBERS

28. Find two numbers such that you get a result between 0 and 1 whether you add them, multiply them, subtract one from the other, or divide one by the other.