1.10 Three Dimensions

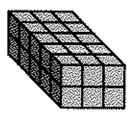
You will need:

the Lab Gear 

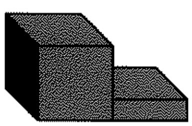
VOLUME

Definition: The *volume* of a solid is the number of unit cubes it would take to build it.

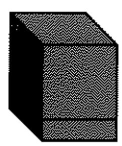
1. What is the volume of this box? Explain how you got your answer.



You can find the volume of a Lab Gear building by just adding the volume of each block. For example, both of these buildings have volume $x^3 + x^2$.

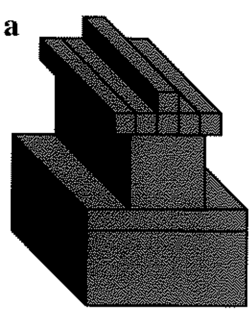


or

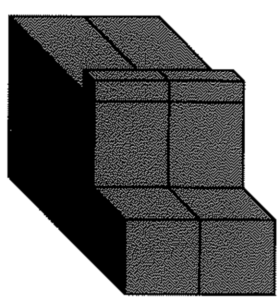


2. What is the volume of each of these buildings?

a

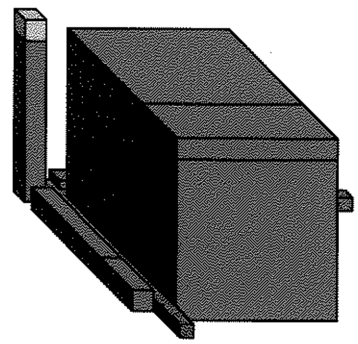


b



MAKE A BOX

Example: This box has volume $y^3 + xy^2 + y^2 + xy$, length $y + x$, width y, and height $y + 1$.



For each problem, the volume of a box is given.

a. Get the blocks.

b. Use them to make a box.

c. Write the length, width, and height.

3. $3xy + x^2y + xy^2$

4. $xy^2 + 2y^2$

5. $x^2y + 2xy + y$

6. $x^2y + xy^2 + xy + y^2$

7. $y^3 + y^2 + xy^2$

8. $x^3 + x^2y + 2x^2 + xy + x$

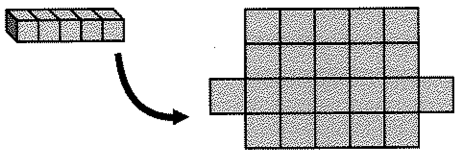
We will return to the volume of boxes in a future chapter.

SURFACE AREA

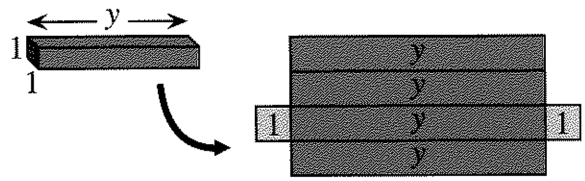
Definition: The *surface area* of a solid is the number of unit squares it would take to cover all its faces (including the bottom).

In simple cases, to figure out the surface area it helps to think of a paper jacket that would cover the whole block. The area of such a jacket is the surface area of the block.

For example, the surface area of the 5-block is 22 cm^2. Its volume, of course, is 5 cm^3.



9. Find the surface area of the 25-block.



The surface area of the blue blocks can also be figured out by thinking of their jackets. For example, the y-block has a surface area of $4y + 2$.

10. Find the surface area of each of the other blue Lab Gear blocks.

DISCOVERY POLYCUBES

Definition: *Polycubes* are obtained by joining cubes together face-to-face. They are the three-dimensional equivalent of polyominoes. Here is a *tetracube.*

There is just one *monocube,* and one *dicube.* There are two *tricubes* and eight *tetracubes.*

All of these polycubes look just like the corresponding polyominoes, except three of the tetracubes, which are really three-dimensional.

11. Find all the polycubes, monocube to eight tetracubes, with your blocks and try to sketch them. Hint: Two of the three-dimensional tetracubes are mirror images of each other.

12. Find the surface area of the polycubes you found in problem 11.

13. Find polycubes having volume 8 and as many different surface areas as possible. There are five different solutions.

14. Were any of your surface areas odd numbers? If yes, check your work. If no, explain why not.

15. For a given number of cubes, how would you assemble them to get the largest surface area? The smallest?

16. What would the largest possible surface area be for a polycube having volume 100?

17. Explain in words how you would find the largest possible surface area for a given volume.

18. For each of the following volumes, find the smallest possible surface area.

a. 12	b. 18	c. 20
d. 24	e. 27	f. 30

19. Explain in words how you would find the smallest possible surface area for a given volume.

MORE ON POLYCUBES

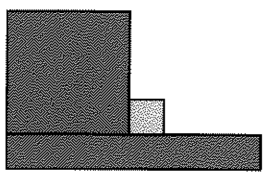
20. Find all the polycubes having volume less than 5. Put aside all the ones that are box-shaped. The remaining pieces should have a total volume of 27. Using wooden cubes and glue, make a set of puzzle pieces out of these polycubes. Assemble them into a 3-by-3-by-3 cube. (This classic puzzle is called the Soma® Cube.)

21. There are 29 pentacubes. Twelve look like the pentominoes, and 17 are "truly" three-dimensional. Find them all and sketch them.

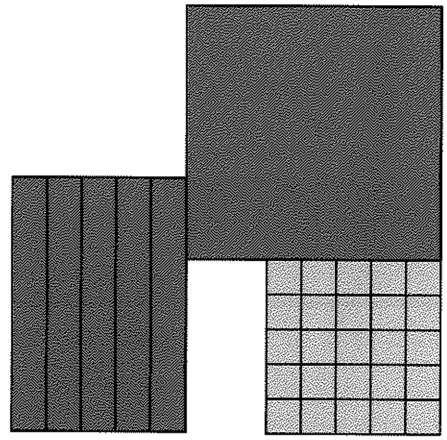
REVIEW ***PERIMETER***

Find the perimeter of each figure.

22.



23.



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