

G L O S S A R Y



Absolute value The absolute value of a number x is the distance from x to 0 on the number line.

Absolute zero The temperature for which the volume (of gases) would be zero—the lowest possible temperature.

Acute angle An angle whose measure is less than a right angle.

Acute triangle A triangle that contains three acute angles.

Adding zero Adding the same quantity to both sides of an equation, or to the plus and minus area on a workmat, is the technique of adding zero.

Area The size of a surface expressed in square units.

Arithmetic sequence In an arithmetic sequence the difference between consecutive terms is always the same. It is called the *common difference*.

Associative Law For all real numbers a , b , and c ,
Addition: $a + (b + c) = (a + b) + c$, i.e., quantities can be grouped in any way.
Multiplication: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, i.e., factors can be grouped in any way.

Average speed The total distance traveled divided by total travel time.

Axis In the Cartesian coordinate system, the horizontal number line is the x -axis. The vertical number line is the y -axis.

Axis of symmetry If the graph of a parabola is folded so that its two sides coincide, the line on which the fold occurs is the axis of symmetry.

Bounce ratio The bounce-height to drop-height ratio.

Cadence The pace of pedaling (a bicycle).

Cartesian coordinate system The Cartesian coordinate system is the technique of using horizontal and vertical axes and graph points to make geometric representations of algebraic equations. It is named for Descartes, the French mathematician and philosopher.

Chunking The process of grouping bits of information into a single piece of information. Also treating an entire algebraic expression as one variable.

Coefficient In a term, the coefficient is the numeric factor of the term or number that is multiplied by the variable.

Commutative Law For any real numbers a and b ,
Addition: $a + b = b + a$.
Multiplication: $ab = ba$.

Completing the square When you add the same quantity to both sides of a quadratic equation (and make a perfect square), you are completing the square.

Complex number A complex number cannot be shown on a number line. It requires a two-dimensional number plane.

Compound inequality An inequality that contains more than one inequality symbol.

Constant A term having no variables.

Constraints A constraint is a condition necessary when solving an equation.

Conversion factor In the case of unit conversion, the proportionality constant (the number by which you multiply) is the conversion factor.

Coordinates In the Cartesian coordinate system, the numbers in an ordered pair, i.e., (x, y) are used to locate a point on a plane.

Degree of an expression The degree of an expression, in terms of the Lab Gear, is the lowest dimension in which you can arrange the blocks.

Density Density is equal to weight per unit of volume.

Discriminant The discriminant is the quantity $b^2 - 4ac$ that appears under the radical in the quadratic formula, sometimes written as the Greek letter delta, Δ .

Distributing the minus sign When you write an equivalent expression without parentheses you are distributing the minus sign.

Distributive Law For any real numbers a , b , and c ,
of multiplication over addition: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$.
of multiplication over subtraction: $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Domain (of a function) The set of values that the input can take.

Dynamic rectangles Dynamic rectangles have the property that half of such a rectangle is similar to the whole.

Equivalent equations If equations in two variables have the same graph on the Cartesian coordinate system, they are called equivalent equations.

Euclidean distance The straight-line distance between two points.

Evaluating expressions When you evaluate an expression, you replace each variable in it by a given value and then simplify the result.

Experiment An example of an experiment would be one roll of a pair of dice. Each different possibility of a result is an *outcome*. An *event* is one or more outcomes.

Exponential growth Involves repeated multiplication by a number.

Exponentiation or Raising to a power The operation of multiplying a number by itself repeatedly. The number multiplied is the *base*. The number of factors is the *exponent*.

Extrapolation When you know data points and use them to predict data values at a later or earlier time, the process is called extrapolation.

Eyes The points of intersection of the grid lines inside a polyomino are eyes.

Factor (noun), Common A common factor divides each term in a polynomial evenly.

Factor (verb) To write as a product.

Fair A game is fair if each of the players is equally likely to win.

Family (of functions) A group of functions that share a certain attribute.

Fixed point If an in-out line is horizontal, its input is a fixed point.

Focus Point where all in-out lines meet, if extended to the left or right.

Function A relation that assigns to each member of its *domain* exactly one member, its *range*.

Gear The gear ratio multiplied by the diameter of the rear wheel (of a bicycle).

Gear ratio The ratio of the number of teeth on the chainwheel (of a bicycle) to the number of teeth on the rear sprocket.

Geometric sequence In a geometric sequence each term is obtained from the previous term by multiplying by a constant amount, the common ratio.

Golden ratio The ratio of the longer to the shorter side of a golden rectangle is the golden ratio.

Golden rectangle A golden rectangle satisfies this property: If you cut a square off one end of the rectangle, the remaining rectangle is similar to the original rectangle.

Group A set of elements, together with an operation, that satisfies certain rules.

Hypotenuse The side of a right triangle that is opposite the right angle.

Identity An equation that is true for all values of the variables.

Inequalities An inequality is a mathematical sentence that contains an inequality symbol between two expressions, e.g. $2 < 6$, $x + 4 > 5$.

Input-Output Tables In such tables, x is the number that is put in, and y is the number that

comes out. Each table has a rule that allows you to get y from x .

Integer Any positive or negative whole number and zero.

Intercepts of graphs

x -intercept: The point where it crosses the x -axis.

y -intercept: The point where it crosses the y -axis.

Intercept form: $y = a(x - p)(x - q)$

Interpolation When you know data points and use them to determine data values between those points, the process is called interpolation.

Inversely proportional You can say that y is inversely proportional to x if the product of x and y is constant. Algebraically, $xy = k$ or $y = k/a$ for some constant k .

Iterating functions To iterate a function means to use its output as a new input.

Lattice line A line having equation $x = b$ or $y = b$, where b is an integer.

Lattice point A point on the Cartesian plane having integer coordinates.

Legs The two sides of the right angle in a right triangle.

Like terms Terms whose variable factors are the same.

Linear combination The equation obtained by adding constant multiples of two equations together.

Magnification In function diagrams that have a focus, changes in y can be found by multiplying the changes in x by a number, called the magnification. Also called *rate of change*.

Mean The average of a set of values.

Median The middle value of a set of values.

Numbers

Rational: A rational number is any number that can be expressed as the ratio of two integers in the form a/b where $b \neq 0$.

Irrational: An irrational number is a real number that cannot be written in the form a/b where a and b are integers.

Natural: Natural numbers are the numbers we count with: 1, 2, 3, 4, ...etc.

Real: Real numbers include all rational and irrational numbers.

Observed probability Can be represented graphically by the slope of the line through the origin and the corresponding data point.

Obtuse angle An obtuse angle is greater than a right angle.

Obtuse triangle An obtuse triangle contains an obtuse angle.

Order of operations A rule for the order in which operations are to be done.

- 1) Compute within grouping symbols;
- 2) Compute powers;
- 3) Multiply and divide in order from left to right;
- 4) Add and subtract in order from left to right.

Origin The point at which the axes of a graph cross; point (0, 0) in the Cartesian coordinate system.

Parabola The graph of a quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$ is a parabola.

Parameter A constant or variable in a mathematical expression which distinguishes specific cases. In $y = a + bx$, a and b are the parameters.

Perimeter The perimeter of a figure is the distance around it.

Pi Pi, π , is approximately equal to the number 3.1415926536. The formula for the area of a circle is πr^2 . (r is the radius of the circle.)

Plaintext The text of a message, before it is encoded.

Polycubes You can create polycubes by joining cubes together face-to-face. Polycubes are the three-dimensional equivalent of *polyominoes*.

Polynomial function A function of the form $y =$ a polynomial.

Polynomials A polynomial is a monomial or a sum of monomials.

Monomial: An expression that is the product of numerals and variables.

Binomial: A polynomial having two terms.

Trinomial: A polynomial having three terms.

Polytans Shapes created by combining tans.

Power A number that can be named using exponential notation.

Power of a product law It states that $x^a y^a = (xy)^a$ as long as x and $y \neq 0$.

Power of a ratio law It states that $x^a/y^a = (x/y)^a$ as long as x and $y \neq 0$.

Prime factorization Prime factorization occurs when you write a whole number as a product of prime factors.

Prime number An integer greater than one that has no factors other than one and itself.

Probability The probability of an event is interpreted to mean the relative frequency with which an event occurs if the experiment is repeated many times.

Product of powers law States that $x^a \cdot x^b = x^{a+b}$ as long as $x \neq 0$.

Pythagorean theorem In all right triangles, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Quadrant In the Cartesian coordinate system, the axes divide the system into four parts, called quadrants.

Quadratic formula A formula for finding the solutions of a quadratic equation $ax^2 + bx + c = 0$. The formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic function A second-degree polynomial function.

Radical

Radical sign: The symbol $\sqrt{\quad}$.

Radical expression: An expression written under the radical sign.

Range (of a function) The set of values the output can take.

Rate of change of a function The rate of change of a function is the ratio between the change in y and the change in x .

rate of change = change in y /change in x

It is often called *magnification*.

Ratio of powers law It states that $x^a/x^b = x^{a-b}$ as long as $x \neq 0$.

Rational expression The quotient of two polynomials.

Rationalizing the denominator Simplifying a radical expression so that there are no radicals in the denominator and only whole numbers or variables in the radicand.

Reciprocals Two expressions are reciprocals if their product is one. Also called the *multiplicative inverse*.

Relative frequency The relative frequency of successes is the ratio of successes to trials.

Repeating decimal A decimal in which the same number or group of numbers repeats endlessly.

Right triangle A right triangle contains one angle of 90 degrees.

Rise The units of altitude gained for every 100 units moved in a horizontal direction (the run).

Run Distance moved in the horizontal direction when dealing with grade and slope.

Scientific notation A number expressed as the product of a power of 10 and a numeral greater than or equal to 1 but less than 10.

Sequence An ordered list of numbers or expressions, called *terms*.

Similarity Two figures are similar if all the dimensions of one can be obtained by multiplying the dimensions of the other by the same number. This number is called the *ratio of similarity*.

Simple radical form Writing the square root of a whole number as a product of a whole number and the square root of the smallest possible whole number.

Simultaneous equations Two or more equations for which you must find a common solution.

Slope A number telling how steeply a line slants; the ratio of rise to run.

Slope-intercept form $y = mx + b$

Solving an equation When you find all the values of a variable that make an equation true, you are solving an equation.

Standard form equation $ax^2 + bx + c = 0$

Step function May be shown by a graph. The end points of the steps may be filled in (closed circles) or hollow (open circles).

Subjective probability Subjective probability is assigned to an event according to a person's own knowledge, beliefs, or information.

Surface area The surface area of a figure (for example, a cube) is the number of unit squares it would take to cover all its faces.

Tan In the world of geometric puzzles, a tan is half a unit square (cut along the diagonal).

Tangent A line that touches a graph at only one point is tangent to the graph.

Taxicab distance The taxicab distance between two points in the Cartesian plane is the length

of the shortest path between them that consists of only horizontal and vertical segments.

Terminating decimal A decimal that can be written in decimal form with a finite number of digits.

Terms An expression that is the product of numerals and variables.

Theoretical probability Can be represented graphically as a line through the origin.

Translations (of groups) A graph obtained by shifting the location of a given graph without changing its shape is called a translation of the original graph.

Variable A letter or other symbol used to represent a number or numbers.

Vertex of an angle The "corner" of a geometric figure is the vertex. The plural is *vertices*.

Vertex form of quadratic function The quadratic function $y = (x - H)^2 + V$ in vertex form.

Volume of solids The volume of a solid is the number of unit cubes it would take to build it.

Zero product property It states that when the product of two quantities is zero, one or the other quantity must be zero.

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