Isomorphism

Definition: Two mathematical structures are isomorphic if their elements can be matched one-to-one in such a way that the results of operations are preserved.

Example: We can match the elements of the YZ group with the elements of the additive mod 6 group as follows:

Y □ 2
Z □ 3
E □ 0

Does this make sense so far? Remember that YYY = E, which corresponds to 2 ⊕ 2 ⊕ 2 = 0. Likewise, ZZ = E, which corresponds to 3 ⊕ 3 = 0. For the isomorphism to work, we need to find the match for the other elements, and make sure the results of the operations are preserved. For example, YY should correspond to 2 ⊕ 2, so YY □ 4.

1. Find the matches for YZ and YYZ. Do we have an isomorphism?
2. There is another isomorphism between those same groups, which starts with Y □ 4. Find the other matches.
3. Is the triangle symmetry group isomorphic to the YZ group? To the yz group?
4. Find an isomorphism between the "def" group of Glosian money and the additive calendar group.
5. Show that the multiplicative calendar group is isomorphic to the additive mod 6 group.

Fields

Definition: An algebraic system {S, +, · } consisting of a set S together with two operations + and ·, is called a *field* if it has the following properties.

∀ a, b, c in S:

A1. Addition is associative: $a + (b + c) = (a + b) + c$

A2. Addition is commutative: $a + b = b + a$

A3. Zero: ∃ an element 0 in S such that $a + 0 = a$

A4. Opposite: ∃ an element -a such that $a + -a = 0$

M1. Multiplication is associative: $a(bc) = (ab)c$

M2. Multiplication is commutative: $ab = ba$

M3. One: ∃ an element 1 in S such that $1a = a$

M4. Reciprocal: if $a \neq 0$, ∃ an element $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$

D. Multiplication is distributive over addition: $a(b + c) = ab + ac$

1. Explain why the integers with + and · are not a field.
2. Explain why the rational numbers with + and · are a field.
3. Show that the set of numbers mod 5 with ⊕ and is a field.
4. Show that the set of numbers mod 6 with ⊕ and is not a field.
5. Is Calendar Math a field?