The Teacher's Role

As the teacher, you are the key to the success of this book. Your role is more that of "the guide on the side" than that of "the sage on the stage." In order to foster independence and self-confidence, you should as much as possible help students by questioning, rather than by telling. When speaking to the whole class, you should model problem-solving techniques and attitudes.

Because the book works very much in a discovery style, you should encourage risk-taking. Students should not be penalized for being wrong when discussing or exploring new material. Being wrong is a necessary part of the process. Conversely, those who venture conjectures should be praised whether or not they are right.

On the other hand, be aware of the fact that unguided discovery of the concepts of algebra is not likely to happen. It took hundreds of years and the world's greatest minds to develop algebra, and your students are not likely to succeed at doing this unaided in the course of one school year. Your leadership is essential. Due to their lack of experience and understanding, student explorations may or may not lead to better understanding. Sometimes students explore and are no clearer on the subject than they were at the beginning.

However, with your leadership, a discovery-based classroom can be an exciting and empowering environment for learning. To provide the leadership that makes this possible, you can

- introduce new topics with pointed questions
- compare various answers to open-ended questions
- encourage students to come up with more solutions to open-ended questions
- help students make connections between different representations of the same concept
- encourage and help the process of generalization
- encourage and help students to summarize what they have learned
- provide hints if students are getting frustrated

Discovering and Investigating

▶ Using Explorations

One way of promoting a discovery-oriented environment in your class is to make good use of the problems labeled Exploration. These problems stimulate exploration of a problem without providing too much structure or guidance. Teachers should provide support and encouragement, or even hints, but should not completely structure that part of the lesson. Explorations are often large in scope and cannot usually be solved by a single student in a few minutes. They are geared to group work, followed and/or preceded by whole-class discussion.

Use your judgment as to how much time to allow for Explorations. On the one hand, it is usually wrong to stop students from exploring a question if they have gotten started and they are continuing to make progress. On the other hand, if students are having difficulty, it may be good to leave the Exploration unfinished and to come back to it after having done more structured work.

If an Exploration goes well (generates enthusiasm, hard work, and interesting discoveries) it may take a whole period. It may also lead to other questions, formulated by the class. Do not expect this, but be open to it.

You may prefer to conduct the Explorations with the book closed, so as to remove the temptation to "look ahead" for a solution. Or you may develop a more flexible style which allows groups who get frustrated with the Exploration to go on with the more structured part of the lesson.

Explorations serve as motivation for the work that follows. By grappling with them, even if they do not completely solve them, students start to understand what the question is that the lesson will answer, and why this particular algebraic question is important or interesting. This helps make the guided investigation that follows more meaningful and effective.

▶ Emphasizing Problem Solving

This book is designed to help students develop algebraic ideas through problem solving. Problem solving is an integral part of the course every day, not a once-a-week adjunct.

To help your students become better problem solvers, you can

- avoid teaching cookbook recipes for solving different types of problems
- bring out different student solutions to the same problem
- discuss whether other solutions exist for a given problem
- lead discussions of difficult problems
- model your own problem-solving strategies
- encourage and model persistence — not all problems can be solved at once.

▶ Using Guided Discovery

While the big problems in the Explorations help put the lesson's question in focus, the small problems in the guided investigation help students take small steps in their understanding of these questions.

In some cases the investigations are simply a guide to one way of solving the preceding Exploration. If your class has been able to go far with the Exploration, you may be able to skip part of the next investigation. Or, you can assign problems from the investigation as homework for reinforcement and to insure that more students get the idea.

Discussing, Explaining, and Practicing

▶ The Importance of Discourse

Part of the reason group work is important is that it allows students to verbalize mathematics. It is by talking that students separate what they understand from what they don't understand. It is by explaining a concept that they deepen their understanding of it. It is by learning to ask the right question that they prepare to understand the answer.

The other vital verbalizing activity is writing. Beyond a certain point, drill loses its effectiveness, as students are turned into unthinking automatons. At that point, they will learn more from writing than from continuing the drill. We often ask students to "explain" their answers. This goes beyond merely "showing work," and can include writing sentences, paragraphs, or drawing sketches. This is not easy for most students, and your coaching and coaxing is particularly important. It is worth the effort, because what they can explain they surely understand.

Because writing is so important, we will discuss it at length a little later.

▶ Reading

We have tried to write a book that students can read. If your students are not used to reading in math class, you will have to insist that they *always* read the problems and their introductions. Some students enjoy reading aloud to their groups, others prefer to read to themselves. Do not discourage either approach. Perhaps group assignments such as Explorations should be read aloud, while other problems are read individually. The important thing is that students read everything.

If you have students who do not know English well, or are weak readers, make sure they get help. Remember: It is more important to learn to read and write than to learn algebra, and math class can help in this process.

▶ Mastery and Skill Development

Because learning is not smooth and linear, students need frequent repetition of important activities, and rethinking about important ideas. Mastery is not immediate. In fact for any important concept, the appearance of quick mastery often masks an underlying lack of understanding. Abstraction is a process that takes time. Rushing it leads to the memorization of poorly understood material as a substitute for understanding.

Our belief is that practice is important, but that it is most effective in the context of worthwhile and interesting work. Even though it may not be obvious at first glance, the lessons of this book entail continuous application of the basic skills of algebra. On the other hand, we have tried to avoid long stretches of repetitive drill. We do provide short bursts of it here and there for those of you who feel that your students need more practice. However we discourage you from using all of the drill at the expense of the more central material.

One way to provide additional practice at a more interesting, less mechanical level is to ask students to create problems of the type you think they need to practice. The process of creating problems for each other does a lot more than solving many problems of the same type, since it requires a better understanding of the problems.

In fact, you should be alert to the fact that the book often asks students to do just that. The basic idea is that students should be able to do everything in every possible direction. For example, they should know

- not just how to simplify expressions, but how to complicate them;
- not just how to solve equations, but how to create equations with a given solution;
- not just how to graph a function, but how to recognize a function from its graph.

This level of understanding is far more meaningful and durable than the mere ability to follow a given algorithm in a narrowly defined situation, which is all that can be gained from repetitive drill.

▶ Homework

This book is written with the expectation that students will have daily homework. In some cases, students can simply move ahead in the text at home. But some work should be done in class, not as homework, for example:

- new material
- material that requires the use of manipulatives
- material that is labor-intensive where the work should be shared

- material that is difficult enough to require group work or teacher guidance

Such assignments are the ones omitted from the *Suitable for Homework* list in the Teacher's Notes. (However, keep in mind that even homework material may need some discussion in class the next day, and that one night's assignment may be more or less than the problems flagged as suitable in a given lesson.)

When you cannot assign the next section for homework, be prepared to assign the Essential Ideas, below-the-line problems, or Thinking/Writing from previous chapters. For more ideas on organizing homework, see below, under *Two Sequences*.

WRITING

▶ Why Write in Math Class?

Writing enhances thinking and learning. When students explain their ideas during group work and class discussion, they have to think more deliberately. Writing requires even more clarity. As students struggle to put their ideas in writing, they have to come to grips with what they don't understand. They find a need for new vocabulary. They look for good examples. They search for the combination of drawings, mathematical symbols, and words that will communicate what they know.

Writing in math class can change the way students think about math as a discipline. Their daily routine in math class in the past might have consisted of providing answers to routine exercises. Both exercise and answer consisted solely of numbers and other mathematical symbols, with the terrifying exception of "word problems." In this book, many of the problems and answers involve a combination of words and symbols. As students work to understand and answer them, their goals shift from simply getting an answer to achieving an understanding. As they write, they summarize, generalize, and resolve contradictions. Writing also gives them the opportunity to raise questions, because as they try to write about a topic they become conscious of gaps in their own understanding. They become more aware of their own thinking.

Writing is not a specialized skill that can be mastered only by the creative few who have a special talent for it. The ability to write in a clear, direct, logical way is a basic skill, and one of the most valued in the workplace. Likewise, teaching writing is not in the purview of the English teacher alone. Writing can enrich your math teaching and add a new dimension to discourse in your class.

▶ How This Book Supports Writing

When students are led by the hand through discovery work, they can sometimes lose sight of the whole idea, even if they competently answer specific questions along the way. They may not be seeing the staircase for the steps. Writing explanations of their work helps students remain alert.

This book is designed so that writing can play a central role in the course. The Key Questions, Summaries, and Generalizations encourage informal, daily writing. They help students see what big ideas should be emerging from all the details. If you are having students build a reference section in their notebooks, entries can be chosen from these problems.

The Reports are intended to be more extensive and formal, but you can vary the format of these to keep students' interest. (See below.) The Thinking/Writing assignments

(each of which contains one or sometimes two Reports) are intended to be used as a major assessment tool. Some teachers occasionally use these like problems of the week, requiring a polished, individual report.

Projects and Research provide other opportunities for larger writing assignments. You will not be able to assign all of these to all students, but they are there to be used as major independent assignments once or twice per semester, or as extra credit assignments. It is nice for students to present what they found to the class, via bulletin boards, or oral presentations. If you are not used to assigning this kind of work or have trouble evaluating it, you can get help from librarians and teachers in other departments. These kinds of assignments help show that math is connected to the world. The sources can be almanacs, newspapers, calls made to businesses or other institutions, visits to the library, whatever. Try to have a few resource books in your classroom, such as an almanac and a dictionary.

▶ Finding a Good Balance

By flagging the problems involving writing, and by providing frequent opportunities for both formal and informal writing, we communicate our belief in its importance. The prominence of the Thinking/Writing assignments allows you to emphasize these lessons as much as chapter tests. Students take writing seriously if they know it is as important a part of their grade as traditional tests. Parents and school administrators who think math class means only symbol manipulation are readily convinced of the power of writing assignments in math when they look at actual student work, and see how these lessons develop competence in problem solving and writing along with understanding of algebraic concepts.

Although we want to encourage writing, students quickly tire of it if they are asked to write too many reports. Varying the format of reports will maintain students' interest and provide an alternate creative outlet for students who find writing difficult. For example, instead of writing individual reports, groups can summarize their findings by drawing and writing with colored markers on posters or butcher paper. These can be displayed around the room and provide a focus for oral group reports and class discussion. Many students are hesitant to speak to the whole class, and doing group presentations on a regular basis will help them develop confidence. Reports need not always be written. You can ask students to prepare oral Reports and Projects, as well as bulletin board displays. Some teachers have even allowed student projects to be video documentaries, computer programs, skits, or dioramas.

▶ Informal Writing

Writing gives teachers a window into the students' minds. By giving students plenty of opportunities for informal writing, we can watch them reflect, analyze, clarify, interpret, evaluate, discuss, question, and conjecture.

Students using this book will do a lot of writing. Almost daily, they will write Summaries or Generalizations, and answer key questions. This informal writing will be in their notebooks, and need not be polished. It is there to help students get familiar with the material and to give you insights on a particular student should you need information beyond that provided by tests and formal writing assignments.

Informal writing can be used frequently as a diagnostic tool at key points in the discussion. You can ask each student (or in some

cases, each group) to spend a few minutes writing. The directions might be quite open, ("Summarize what you learned from this discussion," or "Write down a question you have about our discussion,") or very specific, ("Describe one method for solving linear equations."). You can have students share their results orally, walk around the room for a quick look at several students' papers, or collect them to look at later to get an idea of the class's understanding of a concept. This technique is especially helpful in getting responses from students who are reluctant to speak up in class.

A frequent question from students who used the preliminary version of this book was, "What do they mean when they ask us to explain?" To help students get used to this, you can write student responses on the board or overhead so that they can see examples and compare explanations. Write an explanation and ask what students would add to it or delete from it to make it clearer. Ask the members of each group to compare their explanations and come up with one group explanation. Explaining will soon become a natural part of doing math problems for your students.

Some students resist writing not because they are afraid of writing itself, but because they are reluctant to confront uncertainties in their understanding and reasoning. This is particularly true of students who have always done well in the kind of math class in which the goal was to follow the teacher's step-by-step instructions and get the right answer. "If I can do it, why do I have to explain it?" is a common question at first. If the atmosphere in your class is one of accepting partial understanding and welcoming the questions that arise from them, such students will become more receptive. Informal writing is a record of the work of the class as it moves together toward a clearer understanding. If you value this process, students will soon see writing as natural and central rather than peripheral to the learning process.

Because we explicitly encourage daily writing, it will be relatively easy for you to incorporate other types of informal writing into the course, such as freewriting, learning logs, and journals. For more information on using these techniques, see Countryman (1992).

▶ Formal Writing

Do not be discouraged if students complain at the beginning of the year that, "This is more writing than my English class." As they see the benefits of writing, the complaints rapidly diminish. Most students soon begin to take pride in the quality of their formal written work, and to value the opportunity to revise it. As an assessment tool, writing has the potential to create a more positive climate in the classroom. Students, who often feel that tests reveal what they don't know, welcome the opportunity to communicate in writing what they do know.

A few students will have done extensive writing in previous math classes, particularly if they have done problems of the week. Others will have had little experience writing in math and may be very anxious about it. You need to communicate from the very beginning that

- you have high standards for the finished product, and
- you will provide support for learning how to create a high-quality finished product.

Student groups of four may work well for day-to-day problem solving and working on assignments, but pairs of two often work better when there is something to polish or

publish. In the beginning, the Thinking/Writing assignments guide student writing with a series of questions. Students can work on these for homework, then work in pairs in class the next day, comparing answers. The second night's homework can be to write up the work in a more polished form. The following day, students read each other's work and make editorial comments. The third night's homework is to write the final report. This works well as a three-day homework assignment while you continue to move ahead with new material in class.

Among the reproducible pages at the end of this Teacher's Guide, we have provided some general guidelines for Thinking/Writing assignments. We suggest that you use these guidelines (or similar ones) to get students started. The four stage process described here is one suggestion for how to work through the early writing assignments. You may want to revise it to fit your own needs. Students will also have suggestions for tailoring the process to the needs of your class. As students become more skilled, you may be able to compress this process and spend less class time on it. We have also included a suggested format for the final version of the report. You may want to begin with this format and revise it as you and your class come up with other ideas.

Organizing the Material

▶ Providing Access and Challenge

Mathematical thinking among the students, not any particular activity, tool, or technique, should be the primary goal of instruction in math class. This is valid not only among the top students, but even more so among average and weaker students. All students can think, even the ones whom we have in the past classified as unable to engage in abstract thinking. In fact, a thinking approach, combined with the themes and tools described above, is the best approach to involving average and weaker students.

All algebra classes are somewhat heterogeneous. Some are very heterogeneous. The way to work with a mixed group is not to "teach to the middle," "teach to the top," or "teach to the bottom." Teach to *all* students by mixing the approaches, the tools, and the levels of difficulty. *If the material is varied enough, you will get everyone involved.*

This book was written for heterogeneous classes. We have tried to have something "too difficult" (for some students), and something "too easy" (for some students) in each lesson. We also offer open-ended questions and projects as often as possible. All this means that the course provides *access* and *challenge* at the same time.

We provide something "too difficult" because not every student needs to master every idea in the course. If they did, the course would be too easy for the top students, who would resent it. The top students' needs are as important as those of any member of the class. Besides, they are your crucial allies in a heterogeneous class, the engine that drives discovery lessons, and potentially your colleagues in teaching their peers. You need to keep them (and their parents) satisfied that they are not being hurt by working in a classroom with students who are less talented in math.

We provide something "too easy" because not every problem needs to be profound. There must be some entryway into the ideas of the lesson for all students. Those who cannot get in through one door should have other options. Easy, step-by-step questions are helpful to such students, and in fact benefit the rest of the class as well, by helping them organize and consolidate their ideas.

We provide **open-ended questions and projects** because in these, students find their own level. To the simple open-ended problem *Find two numbers that add up to 10*, one could provide a number of different answers. Similarly, in a project such as the newspaper article in *Instant Riches*, (Chapter 3, Lesson 1) some students will produce a multi-page, coherent, well-illustrated, original, and mathematically sound paper, while others will barely manage to turn in a sketchy paper that rehashes some of the ideas in the text.

For open-ended questions and projects, make sure students get a chance to share their work with their classmates if they wish. This helps enrich all students' experience.

The material in this book can be appreciated at many levels simultaneously. Do not expect all students to get the same thing out of any given lesson. Some ideas should be mastered by most, but others will be grasped by only a few. A good model of your role is that of a sports coach or drama teacher, who encourages each student to perform at the best of his or her ability, without expecting all students to be at the same level.

▶ Two Sequences

This book contains many rich problems on which you could spend a very long time. It would be easy to get involved in solving interesting problems and to discover at the end of the year that you had completed far too little of the course. For this reason, we recommend that you continually forge ahead, and one way of doing this is to organize your assignments in two parallel sequences.

The Forward Motion Sequence: This is the main sequence. It consists mostly of the main body of the core Lessons, some of the Thinking/Writing assignments (the ones mentioned in the Teacher's Notes as necessary to the sequence), and any below-the-line Preview assignments. Much of this work is done in class, and some as homework as needed. All of it needs to be done more or less in the order it appears in the book.

The Review and Extension Sequence: This is the support sequence and is just as important. It consists primarily of below-the-line Review or Discovery material, most of the core Thinking/Writing assignments, and the Essential Ideas.

The Essential Ideas, for example, are almost always suitable for homework, can be used as preparation for an end-of-chapter test or can be saved for later review of the chapter. They need not be assigned right at the end of the chapter and are perfect elements to include in the Review and Extension sequence.

One payoff of the two-sequences approach is that when you get to a section that just cannot be assigned as homework, or when you are engaged in an interesting but time-consuming digression in class, you can assign Review and Extension material for homework. For example, a geoboard lesson at the end of a chapter, which is likely to require more than one day of in-class time, might be followed by a lesson or two involving the Lab Gear at the beginning of the following chapter. This is a good time to draw on the reservoir of Review and Extension assignments as a source of homework assignments.

▶ Pacing

Because of the spiral approach of the book, which incorporates preview of difficult ideas early on, and review long after the ideas are first encountered, it is not necessary (or possible) to wait until everyone in

the class has mastered an idea to move on. Mastery often takes a long time for many students and, especially in a heterogeneous class, it is important to keep constant forward motion. You can use the Essential Ideas to find out what mastery is expected at the end of a chapter.

If mastery is not expected, do not waste time by assigning extra practice from other sources. For example, students will solve linear equations in various ways for six whole chapters before they are expected to have mastered them. Teaching them everything you know about equation solving in Chapter 1 will undermine the work on equations that is built into the next five chapters, and rob the students of worthwhile discoveries.

If you are teaching a one-year Algebra 1 course, be careful not to linger too long on the first three chapters. These chapters are designed to get students involved in problem solving, and they contain many rich, open-ended activities. Teachers who used the preliminary version of this book found that students sometimes got so interested in a problem that they did not want to move on to the next section before they had completely explored it. If you have more than a year to teach the course, or are using the first part of it in Pre-Algebra, you can use this feature to your advantage to promote problem solving and encourage the practice of delving deeply into problems. However, in a one-year course, you will have to make some compromises, so don't spend more than about two and one-half weeks on each of the first three chapters.

▶ Challenge and Enrichment

Do not routinely skip challenge (light bulb) problems. A small amount of frustration is not bad, and success at these problems can be thrilling. Make clear that you do not expect everyone to be able to do these problems, but that they should be tried. They are important for the stronger students to work on and are well suited to group work.

If you are concerned that your top students are not being challenged enough, there is plenty of optional material you can use as "extra for experts."

▶ What to Skip

It is not likely you can "cover" this entire book in one year with an algebra class. Do as much as you can, but remember that your goal is to help students *uncover* and *discover* material, not "cover" it. There's a whole ocean of mathematics out there — all you can do is take the students to the beach. At this level, generating interest and confidence in algebra is more important than mastering particular algebraic topics.

To help you select what to skip, and as a first approximation, we have marked some material as optional by displaying it on a light beige background.

However this is only an approximation. What we included in the optional work is

- some less traditional topics
- some time-consuming problems
- some particularly difficult problems
- some puzzles
- some material that previews future courses such as Geometry, Algebra 2, Pre-Calculus, and Abstract Algebra

You need not agree with us on these choices, and you should feel free to make your own decisions. In particular you should skip work that seems to provide repetitive drill in areas where your students do not need more practice.

You will have to use your judgment about how much time you can devote to optional activities and when you do them. They are often the least traditional part of the book, but they can be the most interesting and valuable. As you get more familiar with this book and its approach, and as the math reform process leads to changes in the list of what you must "cover," you can use more and more of them. In our own classes, we intend to use as much of the optional work as we can.

The best overall guideline on skipping is to skip material you don't enjoy teaching. However, be cautious about skipping core lessons. The most likely core lessons to skip are the ones that seem to be hammering an already-familiar topic into the ground. You may also be tempted to skip lessons that you yourself do not fully understand. However, remember that the best way to learn something is to teach it.

▶ A Use for Skipped Material

A good use for optional material you do skip is to assign it to speed demons, those students who want to "get ahead in the book." Getting ahead should be discouraged, because it makes students less available to their group, and because it often leads to superficial understanding. Instead, demand more depth from these students: higher-quality work on reports and projects and extra work on optional material. (Of course, assign material to them that you are sure you will not want to assign to the whole class later.)

Creating a Community of Learners

▶ Student Involvement

We need to get away from the image of the student as passive receptor of information, and the teacher as the source of the information. Teacher explanation, no matter how clear and patient, does not usually succeed in getting across difficult ideas. Instead, we need to think of the student as explorer, collaborator, question-poser, evaluator of the validity of statements, and peer teacher. Everything we do must put student involvement at the center, because math is not a spectator sport. One learns math by doing math.

▶ Groups of Four

Groups of four is an arrangement that maximizes student involvement. This does not mean that all work needs to be done in a group, but it does mean that students should routinely sit in such a way as to be able to work in a group.

You probably have your own way of selecting groups. Teacher-organized groups (as opposed to randomly selected ones) can backfire. Students are quick to jump to conclusions like, "I am the stupid person in this group," or "I am the smart person in this group," neither of which is helpful.

You may try random groupings that change every two to four weeks. Random groupings are sometimes homogeneous, and sometimes heterogeneous, which allows you to take advantage of both types of arrangements. In a heterogeneous group the stronger student can take the lead, or serve as a resource. In a homogeneous group of stronger students there is the excitement of being able to do really fancy work fast. In a homogeneous

group of weaker students there is the comfort of not "feeling stupid" and not having to compare oneself to some star student. As long as the groupings are temporary, the students do not feel trapped, even if they do not like some of the students in the group.

▶ Cooperative Learning

Like most algebra books, this book is probably too hard for an average student to work through individually. However, it is not too hard for an average or even weak student working in a cooperative learning group in the presence of a teacher.

There are many approaches to cooperative learning, and we cannot present them all here. We have had success with an informal approach.

Students work individually much of the time, but can ask each other for help whenever they need or want to.

Large or tedious tasks can be split between members of the group.

Difficult problems can be read aloud, then discussed.

Class can start each day with groups going over their homework.

The purpose of the math class is not to teach cooperation. Rather, the purpose of cooperation is for the students to learn math. Do not expect the students to start the year knowing how to work in groups. Give them as much structure as necessary to help them get off the ground, but don't spend too much time discussing how groups *should* run.

Instead, circulate among the groups and use modeling or direct intervention to redirect the group in the direction you want. For example, if one student is being left out, involve him or her in the work. If a capable student is not being asked for help, or is reluctant to give it, you can ask him or her to answer a tricky question someone else is struggling with. If a group is losing focus, get them back on task, and so on.

We have found that external rewards for effective group work are less necessary if,

- group work is used only for activities for which it is well suited rather than for activities which are better done individually;
- students understand the goal of group work is to help them learn math, not fit some incomprehensible teacher agenda;
- group work is part of the department and school culture.

Certain problems are especially suited to group work, perhaps because they are open-ended Explorations, or because a lot of data need to be collected. Such problems are mentioned in the Teacher's Notes. Finally, it is sometimes useful or necessary to have students work in pairs, for example when working with the graphing calculators or the Lab Gear.

▶ Whole-Class Discussions

You will find that sometimes the groups cannot handle some part of the work. A problem may be too difficult, a concept may have been insufficiently clarified, and one or more groups grind to a halt. Or, you may feel the class needs a change of pace. At such times, it is good to have a whole-class, teacher-led, discussion. The idea is to lead the class to a discovery or understanding through well-asked questions.

At times you may even use a mini-lecture to explain a particularly tricky idea. The key is for your lecture to answer questions your students have as a result of working on

problems. Then, students will listen. Answering questions they don't have is a waste of everyone's time, as they are not likely to listen to or absorb what you have to say.

It is not wrong to start a class with a lecture or chalkboard demonstration, especially if it concerns real questions based on the previous day's work. (This is sometimes necessary if you have trouble getting the class back to a single point of focus once group work has started.)

Keep in mind that when you ask questions of the class, you cannot judge what the whole class is thinking by the answers of a few. This is one of the main limitations of whole-class work, but it can be compensated for periodically by making it a point to ask all students to write down answers to a question, or to ask a question for group discussion.

▶ Individual Work and Responsibility

The purpose of group work and of whole-class work, is to develop mathematical power *in the individual*. Students should generally do at least their homework and quizzes or tests individually. Of course, it is fine to have exceptions to this, but make sure that students have some work they do on their own every day. This will allow them to "know whether they're getting it." In fact, there should be a record of all work in each student's notebook, even when the work is done collaboratively.

Much of the work in this book can be done by students individually. Certainly most of the work labeled *Suitable for Homework* fits in that category. However, when working on new ideas or challenging problems, students should be able to ask other members of their group for help (except during tests)!

▶ Dealing with Absences

Absences are more difficult to make up in a class where cooperative work and the use of tools is essential. Here are a few suggestions.

Make a set of Lab Gear blocks and a geoboard available for checkout from the school library.

Allow students who have been absent to borrow your Teacher's Edition to correct their work.

Arrange for peer tutoring, by members of their group, for students who have been absent.

Creating an Algebra Laboratory

This book provides the basis for turning your classroom into a laboratory for learning algebra. Many students will come into your class unaccustomed to the idea of using tools to learn math. Adolescents can be a conservative bunch, and some, particularly those who are good at memorizing and following rules, will resist the new approach at first. Teachers who used the preliminary edition found that a slight rearrangement of their classrooms to promote the notion of laboratory learning not only enhanced their teaching but helped eliminate student resistance as well.

These teachers placed manipulatives and other materials, such as scissors, graph paper, and dot paper, on shelves around the room, making them easily accessible to students. Colored markers and butcher paper were available for making group reports. When possible, graphing calculators were also available at all times. After a couple of months students had become familiar with all the tools and began to use them spontaneously, even when the lesson did not specifically call for them. Some even chose to use the Lab Gear to check their work on tests.

▶ Calculators and Computers

The only electronic tool absolutely required for this course is a calculator. If your students cannot afford to purchase their own calculators, you should try to secure funding for the purchase of a class set.

A graphing calculator is ideal. If these are not available, you can use any calculator which has the following capabilities:

- negative numbers ([+/−] or [(−)])
- squaring ([x^2])
- powers of ten, scientific notation ([10^n] , or [EXP] , or [EE])
- reciprocals ([1/x] or [x^{-1}])
- exponentiation ([y^x] or [x^y])
- square roots ([$\sqrt{}$]) and ([π])

The ability to deal with fractions is a major advantage ([$a^b/_c$]), but trigonometric, log, and exponential functions are not needed.

There are many lessons in this book that are greatly enhanced by the use of an electronic grapher. The ideal situation is to have a graphing calculator for at least every pair of students. If these are not available, having students work with graphing software in a computer lab or having a teacher-run demonstration with a computer can be effective alternatives. If you have no access at all to an electronic grapher, students can do the graphing by hand, sharing the calculations and graphing, so that they can still see a number of graphs without the tedium of constructing every single one. Remember in this case that the goal is to see and use the graphs, not to become skillful at constructing them.

We are careful not to create situations where technology is used for its own sake. The lessons involving graphing are constructed to insure students have had some experience doing pencil-and-paper graphing so that they understand the concept before using a graphing calculator. We also give you suggestions in the Teacher's Notes about where to introduce the graphing calculator.

A few lessons in the book would be enhanced by the use of a computer spreadsheet, a grapher that will graph lines given in the form $ax + by = c$, or simple statistical software. We indicate this in the Teacher's Notes, but once again you will find that these technological enhancements, while desirable, are not necessary.

If you have access to graphing calculators or computers, but are reluctant to use them because of lack of familiarity with new technology, remember that your students are your allies. Many of them are more technologically literate than their teachers, and you can use this to your advantage. One way of challenging bright students who finish early is to ask them to learn how to use the calculator or computer to do something that is needed for a future lesson. When they have learned, they can teach you and the rest of the class.

In one class that used this book, two students discovered how much time they could save by using the programming capabilities of the graphing calculators. They had soon taught the entire class. This example illustrates the importance of encouraging students to use tools as much as they like, and in their own way. In this book, we make suggestions only for their use. As you and your students become more familiar with them, you will discover many more uses.

▶ The Algebra Lab Gear®

Each pair of students will need one set of Lab Gear. In most cases it is best for students to work with their partner. However do not allow students to avoid work by letting their partner do all the manipulation of blocks, or conversely, all the record keeping. Students should take turns so as to develop competence in all aspects of the work.

It is, of course, important that students respect the materials. A little time must be allotted at the end of each period for putting the Lab Gear away. Improper use of the blocks, such as throwing them, should not be tolerated. Some teachers threaten students who misuse the blocks with having to do all the Lab Gear problems without blocks, instead sketching what they would do with them.

Sometimes, you will need to get the whole class to pay attention to a discussion at the overhead. At such times, ask students to have "no plastic" in their hands. (We tell our students that scientific studies have shown that plastic in student hands impairs their ability to hear.) Students who want to explain something to the class can come up to the overhead projector and demonstrate, or can talk while you demonstrate.

Keep in mind that the Lab Gear is not intended to be used as a mechanical "do-as-I-do" representation. Instead think of it as a creative medium, which can be used by you and your students to explore, model, and illustrate many algebraic ideas. You and your students should feel free to question the way the Lab Gear model is presented in the textbook and to create your own variations on it or extensions to it.

▶ Student Resistance

While we have found the Lab Gear to be popular with a wide range of students, there are a few who do resist it. If this happens, keep in mind that the manipulatives are a means, not an end. If students can demonstrate competence with the ideas, it is not essential that they use the Lab Gear. However, most students will gain from using it, if only when communicating with their classmates. Some encouragement on your part at the beginning of the year may help them overcome their anxiety. Often the biggest resistance is before students have had a chance to use the Lab Gear, before the blocks actually grow on them.

The biggest factor determining whether students will enjoy the work with the Lab Gear is your own attitude. Do not use the blocks in cases where you do not think there is much to be gained, because your ambivalence will be transmitted to your students. Use the blocks only if you are comfortable with the lesson. On the other hand, do not promote the use of the Lab Gear as "fun." Students may wonder, "Fun, compared to what?" The use of manipulatives is intended to provide an arena for tackling difficult ideas. It is a part of learning algebra, not a break from it.

Later in the year you can be more flexible and acknowledge individual differences by allowing those students who still need it to use the Lab Gear, while letting those who don't to work just on paper. The student who can execute the algorithms, but cannot model them with the Lab Gear, does not have as full an understanding as the student who can do both, though he or she may be able to manage well enough.

▶ The Lab Gear® as a Map

When in a new city, to get from one place to another, one may rely on instructions like these: "...take 80 south to the bridge, cross the bay, then take 101 north. Get off at Fell...(etc.)." The problem with this approach in a new city is that even if you remember the instructions, a single mistake along the way means you're lost. If you miss a turn, you may look up at the street signs and not recognize them. At that point all you can do is ask someone for help.

Being able to use a map is a more effective method of finding your way. It allows you to choose among alternate routes, for example, if road work is being performed somewhere along the way.

The usual algorithm-driven way to teach algebra corresponds to the giving of directions. "First you get rid of the fractions, then...(etc.)." Students who get lost have no recourse but to ask the teacher for help. The Lab Gear constitutes a map of much of the territory. Students can use it to check the validity of a statement, or as a strategy to solve a problem, without having to resort to asking the teacher. Like learning to use a map, there is a certain initial investment of effort, but it pays off in increased independence and self-confidence.

The eventual goal is to know the city of algebra well enough to find your way without a map or directions. Someone who already knows algebra has no need for the Lab Gear or for following any particular algorithm. The aim is for our students to get to the point where they just *know* their way around elementary algebra and its applications.

ASSESSMENT

▶ Grading or Assessment?

Assessment and grading are not synonymous. In order to teach a class effectively, you need to have a sense of where the class is as a whole. You also need warning flags about students who may be having trouble with the material. Informal assessment of the class as a whole, of groups, and of individuals, should be going on at all times.

Observing group work is a very effective way to see where students are. Work with the Lab Gear in particular, being extremely visual, allows you to spot immediately the students who are having trouble with the material. When new groups are formed, you should try to identify in the first day or two which ones will need most attention and support from you and make sure to spend more time with them.

You can periodically ask groups to make impromptu presentations to the class on what they have discovered. This can take the form of an oral presentation, but it usually works better if it is accompanied by supporting materials in the form of student-created transparencies or butcher-paper-and-marking-pen posters.

When conducting a whole-class discussion, make sure to ask students frequently to work problems on paper. Then circulate around the room to see their work. Finally, you can use random checks of student notebooks as yet another informal assessment method.

The move toward more writing in all parts of the curriculum is part of a larger movement to encourage and assess authentic achievement. There is a variety of formal assessment methods that you can use to

assess student understanding and to help students monitor their own progress. These are designed to be integral to the learning process, rather than simply a source of grades. In the following sections, we will describe some of these in more detail.

If it is at all possible in your school culture, you should gradually move away from an over-emphasis on grades as the motivator, and add as much as possible to the part of the work students do without external rewards. Much of the work students do should not be graded on the basis of "right" and "wrong." Specifically:

- participation in class discussion,
- participation in group work,
- reliably doing one's homework,
- keeping a neat and organized notebook.

All these behaviors are essential to a student's success in the course, and should favorably affect the grade, *no matter whether the work is correct*. Because mistakes, partial understandings, and confusion are a normal part of the learning process, they should not be penalized.

Not everything needs to be graded. On the other hand, the goal of the course is competence in algebra, and parents, students, colleagues, colleges, and society in general expect that to be measured with grades. To arrive at those, you can use a combination of reports and tests, as described below.

▶ Evaluating Reports

Writing assignments are a very important part of a fair grading policy, because they give you a chance to evaluate students' understanding without some of the limitations of tests. (See below.) It is important to establish high expectations early. Although you may be disappointed with the quality of students' written work at first, do not give up. If you give students the message that reports and projects are just as important as tests, and you assign them on a regular basis, you will find that most students will improve. This requires you to start with small assignments, and perhaps to ask for rewrites, but the extra effort is worth it in the long run.

Formal writing assignments are essential, but be realistic in the number you assign, as they are time consuming for both student and teacher. As a rule of thumb, plan on using one large writing assignment every three or four weeks if you give in-class tests frequently, or every two weeks if you do not.

Using a holistic scoring method will enable you to assign more writing without being inundated by a sea of papers. Most teachers begin by first quickly dividing papers into three piles: *exceeded expectations*, *got the point*, and *missed the point*. Among the reproducible pages in the back of this Teacher's Guide, we have included a *Writing Assignment Evaluation*. Students need to know from the beginning how their work will be evaluated. For more ideas on rubrics and holistic scoring see Pandey (1991) and Stenmark (1991).

▶ Tests and Quizzes

Some reformers have advocated completely eliminating tests, but we disagree. Tests and quizzes have three important functions.

- They help students know what they know.
- They help you know what students can do by themselves without the help of a group.

- They send the message that the purpose of the group is to help everyone learn, not to allow some people to loaf while others work. In the end, everyone is individually accountable.

On the other hand, tests and quizzes should not be overemphasized because, for many students, performance under time pressure is not an accurate reflection of their ability.

Be sure that the tests and quizzes you give reflect the skills being developed in this book. Great speed in computation and accuracy in algebraic manipulation should not be emphasized to the detriment of such skills as using graphing to solve problems, writing clear explanations, and so on. Tests should evaluate students' ability to use what they learned, not primarily their ability to remember facts. You should also use some open-ended questions on occasion. Calculators and other tools should definitely be allowed.

The Essential Ideas can be used as a source of ideas for a chapter test.

It is a good idea for tests to be cumulative as much as possible. One approach is to keep bringing back on tests new versions of problems that proved to be difficult in previous tests. This very effectively gives students the message that they'd better learn it.

Another variation on the traditional test is to ask students to do a Thinking/Writing assignment, minus the Report, in preparation for the test. The test then includes questions of the type that would have been answered in the Report, and students have to write detailed answers.

Note: A set of reproducible tests, with solutions included, is available from Creative Publications. Please see a current Creative Publications catalog for ordering information.

▶ Notebooks

All students in this course should keep a notebook. Since many definitions and generalizations are developed by the students themselves, it is essential that they keep an organized written record of their work.

Enforcing neat notebooks is difficult at first, but well worth the effort. It is best to decide exactly what format you want for the notebooks and insist that students follow it. Otherwise, many will keep a notebook that is much too chaotic to be of much help to them. (A sample notebook assignment with a suggested format appears among the reproducible pages at the end of this Teacher's Guide.)

Notebooks can be checked periodically. One method is to pick one or two names at random every day and check those students' notebooks. Let the students know that their notebooks are part of what they are being evaluated on. Since the notebook contains a record of almost all of a student's work, it can be used to check on many things, such as:

- completeness of homework
- understanding of certain ideas (especially by looking at the problems marked *Useful for Assessment* in the Teacher's Notes.)
- quality of the writing: The "Reports" section of the notebook can serve as a ready-made portfolio of student writing. (You may ask students to indicate which reports reflect their best work and make sure to look at those.)

▶ Portfolios

A portfolio is a selection of student work that gives a comprehensive summary of a student's work in the course. Portfolios provide another alternative for assessment.

You might check notebooks merely for organization rather than content. They can be thought of as working portfolios in which students keep a record of all their work.

Every two to eight weeks, depending upon how significant portfolios are as a part of your assessment program, students should choose material from their notebooks and create a portfolio for assessment. You may wish to choose some of the assignments to be included, but at least some of them should be chosen by the student. Students should also be encouraged to revise some of the work they choose to meet a higher standard. The portfolio should provide not only a chance to show their best work but also a record of their progress. (Because of the importance of showing progress, all entries in a portfolio should be dated.)

For assessment, portfolios can be far superior to notebooks for several reasons. First, they put more responsibility on the student. Students generally have some choice in what is included in the portfolio. Portfolios provide a record of student progress. Students are required to look over their work, to reflect on what they have done well and what needs improvement, to select what represents their best work, and to provide evidence of progress.

Many teachers ask students to include a cover letter each time they add a new set of papers to their portfolios. The cover letters may include a brief description of the items in the portfolio, perhaps accompanied by an explanation of why they where chosen. They may also include a summary of the main concepts covered during the time period. The cover letters give students a chance for self-assessment, and their reflections on their progress are often very revealing. Self-assessment will be new for many students, and it may take some time before they can be honest and thoughtful about it. Be patient and encouraging.

At the end of the semester, you may want students to cull their portfolios and put together semester portfolios. Some teachers allow students to use their portfolios as a reference during the final exam.

As with written reports, the standards by which portfolios will be evaluated should be clear to the students in advance. Some possible criteria for evaluating portfolios can be found in the reproducible pages in the back of this Teacher's Guide. You will want to discuss criteria with students and revise them as the year progresses. We suggest that you use only a few criteria each time you look at portfolios. For example, you may want to concentrate on how students are using different problem-solving strategies and ask them to choose items for their portfolios to illustrate that they can use more than one strategy.

For more information on using and evaluating portfolios, see Stenmark (1991).

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