LESSON 13.7

Finding the Vertex

You will need:

graph paper

graphing calculator (optional)

In this lesson you will learn how to find the vertex of graphs of quadratic functions. This will help you solve quadratic equations.

TRANSLATING A PARABOLA

Graph these functions on the same pair of axes. Use graph paper, even if you have a graphing calculator. For each one:

a. Graph the parabola.

b. Indicate the axis of symmetry with a dotted line, and label it with its equation.

c. Label the vertex with its coordinates.

1. $y = x^2 - 5$ **2.** $y = x^2 - 4x + 4$

3. $y = x^2 - 4x - 1$

Definition: The graphs obtained by shifting the location of a given graph without changing its shape are called *translations* of the original graph.

The graphs you drew in problems 1 through 3 are all translations of the graph of $y = x^2$.

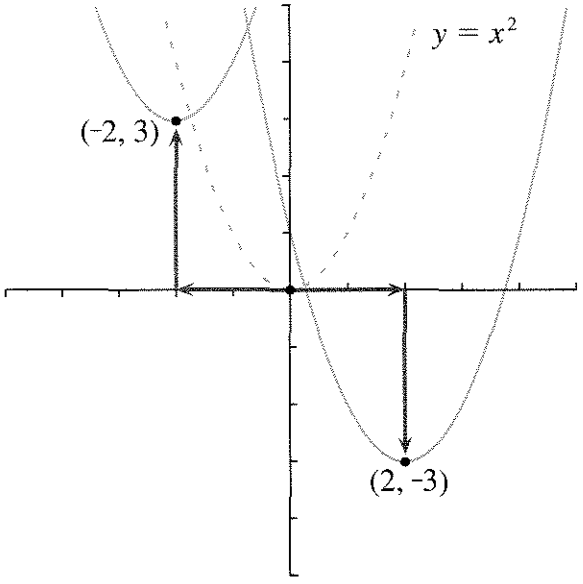
4. Which of the graphs you drew in problems 1 through 3 was obtained by shifting $y = x^2$

a. horizontally? b. vertically?

c. both horizontally and vertically?

VERTEX FORM

The vertex of the graph of $y = x^2$ is (0, 0). When the graph is shifted, the vertex is (H, V).



- If V is positive, the parabola $y = x^2$ has been shifted up; if V is negative, it has been shifted down.
- If H is positive, the parabola $y = x^2$ has been shifted to the right; if H is negative, it has been shifted to the left.

The graph of each function below is a translation of $y = x^2$. For each function:

a. Make a rough sketch of the graph.

b. Show the translation with arrows, as in the preceding figure.

c. Label the vertex with its coordinates.

(If you have a graphing calculator, use it for these problems. However, you should record the graphs with sketches on graph paper.)

5. $y = x^2 + 4$ **6.** $y = (x - 6)^2 - 4$

7. $y = (x + 6)^2 - 4$ **8.** $y = (x + 6)^2 + 4$

9. $y = (x - 6)^2$ **10.** $y = (x - 6)^2 + 4$

11. Write the equation of a parabola that is a translation of $y = x^2$ and has

a. a vertical distance of 8 and a horizontal distance of -3 ($H = -3$, and $V = 8$);

b. a vertical distance of -4 and a horizontal distance of 5;

c. 6 units to the left and 5 units down;

d. 3 units to the right.

Earlier in this chapter you looked at equations of parabolas having the form $y = a(x - p)(x - q)$. That form was convenient for finding x-intercepts.

12. Explain why the equations in problems 5-10 are in a form that makes it convenient to find the vertex by just looking at the equation.

The quadratic function $y = (x - H)^2 + V$ is said to be in *vertex form.*

13. Explain why the H in the vertex form equation is preceded by a minus, while the V is preceded by a plus.

14. The graph of $y = x^2$ meets the x-axis in one point. Give examples of translations of $y = x^2$ that meet the x-axis in the given number of points. Include explanations of how you chose different values of H and/or V.

a. 0 points b. 1 point

c. 2 points

SITTING ON THE x-AXIS

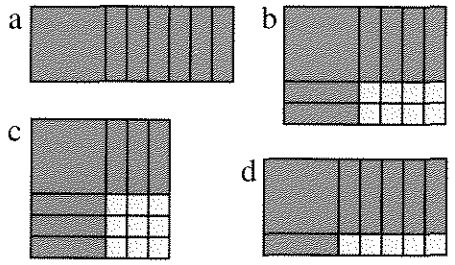
The quadratic function $y = x^2 + bx + c$ is said to be in *standard form.*

For problems 15-21, consider these five equations:

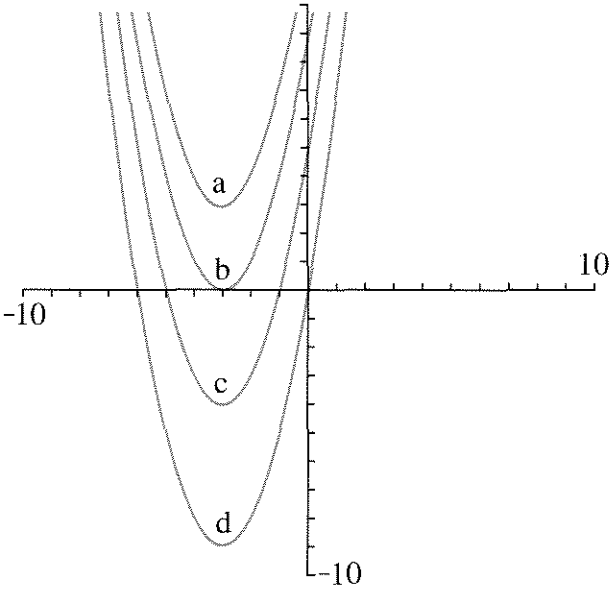
$y = x^2 + 6x$ $y = x^2 + 6x + 5$

$y = x^2 + 6x + 8$ $y = x^2 + 6x + 9$

$y = x^2 + 6x + 12$



15. Match each Lab Gear figure with an equation from the list of five.



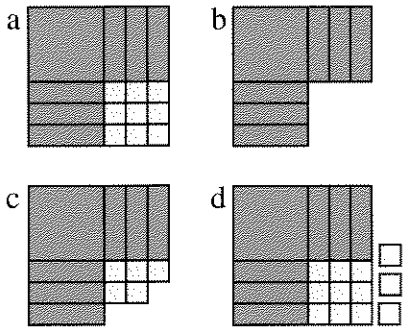
16. Match each parabola with an equation from the list of five.

17. Explain how to identify the parabolas with the help of:

a. the y-intercepts;

b. the Lab Gear figures, combined with the x-intercepts and the zero product property.

18. Explain why the graphs of perfect square quadratic equations have their vertices on the x-axis. (Hint: What is V?)



19. Match each Lab Gear figure with the corresponding equation from the five given earlier.

20. Find V for each equation of the five.

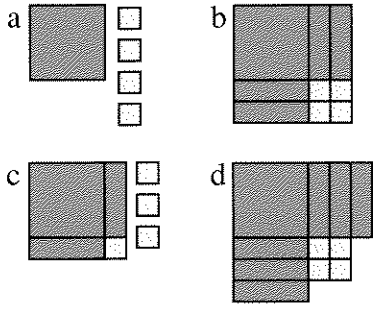
21. Explain how you can find V,
a. by looking at the Lab Gear figure;
b. by looking at the equation.

STRADDLING THE y-AXIS

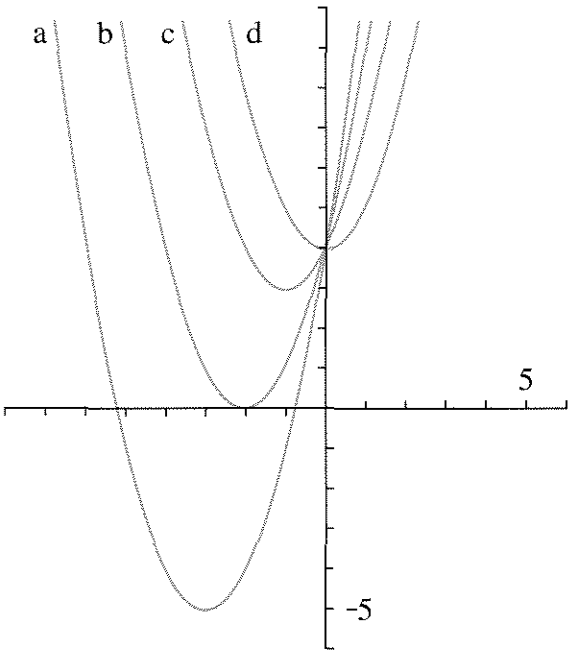
For problems 22-27, consider these four equations:

$y = x^2 + 4$ $\qquad$ $y = x^2 + 2x + 4$

$y = x^2 + 4x + 4$ $\qquad$ $y = x^2 + 6x + 4$



22. Match each Lab Gear figure with the corresponding equation from the four.



23. Match each equation with the correct graph.

24. Explain why the graphs of equations of the form $y = x^2 + c$ have their vertex on the y-axis. (Hint: What is H?)

25. Find H for each equation in the list of four.

26. Explain how you can find H,
a. by looking at the Lab Gear figure;
b. by looking at the equation.

27. a. What is H for any graph of an equation of the form $y = x^2 + 16x + c$?
b. What is H for any graph of an equation of the form $y = x^2 - 16x + c$?

28. **Generalization** Explain why, for graphs of equations in the form $y = x^2 + bx + c$, $H = -(b/2)$.

29. **Report** Write an illustrated report explaining how to find the vertex of a parabola if the equation is in:
a. the form $y = (x - p)(x - q)$;
b. vertex form; c. standard form.