LESSON 12.7

Iterating Linear Functions

You will need:

- graph paper
- or function diagram paper

TREE HARVESTING

Paul's Forestry Products owns two stands of trees. This year there are about 4500 trees in Lean County and 5500 in Cool County. So as not to run out of trees, the yearly harvesting policy at each location is to cut down 30% of the trees and then plant 1600 trees. For example, in Lean County this year they will cut 1350 trees and plant 1600 trees.

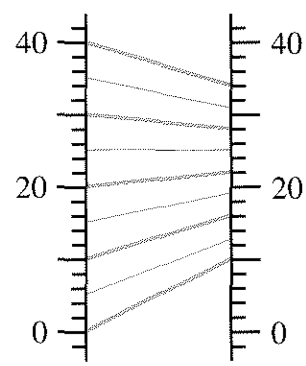
1. Make a table of values showing how many trees they would have at each location every year for nine years.
2. Describe the change in the number of trees at each location. Is it increasing or decreasing? Is it changing at a constant rate from year to year? What do you think will happen in the long run?
3. Write a formula that would give the number of trees next year in terms of the number of trees this year. (Use y for next year's number and x for this year's number. What you get is called a *recurrence equation*.)
4. How many trees would they have at each location after 30 years?

DRUGS

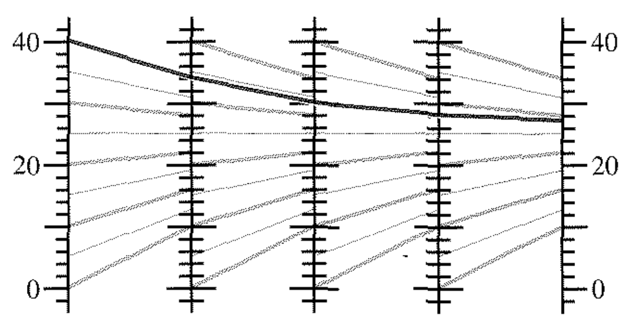
To control a medical condition, Shine takes ten milligrams of a certain drug once a day. Her body gets rid of 40% of the drug in a 24-hour period. To find out how much of the drug she ends up with over the long run, we can use function diagrams.

5. If x is the amount of the drug Shine takes per day, and y is the amount that ends up in her body over the long run, explain why the recurrence equation is $y = 0.6x + 10$.
6. Make a table of values for the recurrence equation, using these values for x, the daily dose: 0, 5, 10, 15, 20, 25, 30, 35, 40.

Here is a function diagram for the recurrence equation.



This function diagram can be repeated to show what happens over the long run. The linked diagrams show how the y-values for one become the x-values for the next.



7. Use the diagram to predict what happens in the long run if Shine takes 10 mg a day of the drug after an initial dose of:
   a. 10 mg; b. 25 mg;
   c. 40 mg.
8. Check your predictions by calculation.

Remember that instead of linked diagrams like in the figure, you could use a single function diagram of the function. Just follow an in-out line, then move horizontally across back to the x-number line; then repeat the process, using the in-out line that starts at that point.

SAVINGS

Glinda puts \$50 a month into a savings account paying yearly compound interest of 6%.

9. What is the interest per month?

10. How much money will she have at the end of one year?

11. Write a recurrence equation for problem 10, expressing the amount in the account at the end of each month as a function of the amount the previous month.

12. Make a function diagram.

13. How does what happens in the long run for problem 10 differ from the problems in the previous sections? Explain.

Definition: To *iterate* a function means to use its output as a new input.

All the problems in this lesson involve iterating linear functions. We will use function diagrams and algebraic symbols to get a more general understanding of this kind of problem.

14. Describe the difference between function diagrams for $y = mx + b$ for the following:

a. $0 < m < 1$ b. $m = 1$

c. $m > 1$

THE FIXED POINT

Definition: A fixed point of a function is one in which the output is the same as the input.

Example: For the function $y = 7x - 12$, when the input is 2, the output is also 2.

15. What is the fixed point for each of the functions in problems 3 and 5? Why was it important in understanding the problems?

16. Find the fixed points.

a. $y = 3x - 6$ b. $y = 3x + 5$

c. $y = 3x$ d. $y = x$

e. $y = x + 3$ f. $y = x^2 - 2$

17. Function diagrams may help you think about these questions.

a. There is a linear function that has more than one fixed point. What is it? Explain.

b. What linear functions have no fixed points? Explain.

18. Generalization

a. Find a formula for the fixed point for the function $y = mx + b$. (Hint: Since the output is the same as the input, substitute x for y and solve for x.)

b. Explain why $m = 1$ is not acceptable in the formula you found. What does that mean in terms of the existence of the fixed point for equations of the form $y = x + b$?

ANALYZING THE SEQUENCES

When iterating a function, you get a sequence of numbers.

19. Exploration Start with the equation $y = 2x + 3$. Change one number in the equation so that when iterating the function, starting with any input, you get

a. an arithmetic sequence;

b. a geometric sequence;

c. a sequence where the values get closer and closer to a fixed point.

Compare your answers with other students' answers.

20. Generalization When iterating $y = mx + b$, different things may happen, depending upon the value of the parameters. Find the values of m and b which lead to the following situations:

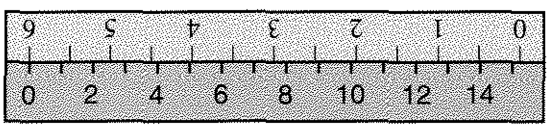
a. arithmetic sequences;
b. geometric sequences;
c. sequences where the values get farther and farther from the fixed point;
d. sequences where the values get closer and closer to the fixed point.

21. Report Summarize what you know about iterating linear functions. Include, but do not limit yourself to these topics.

- real-world applications
- use of function diagrams
- the fixed point
- these special cases:
  - ◊ $b = 0$
  - ◊ $0 < m < 1$
  - ◊ $m = 1$
  - ◊ $m > 1$

DISCOVERY *TWO RULERS*

Alice's ruler



Oliver's ruler

Alice and Oliver lined up her inch ruler against his centimeter ruler, as in the above figure. This yielded the following table of numbers.

x	y
0	6.0
2	5.2
4	4.4
6	3.6

22. a. Graph these data.
b. What is the equation for y in terms of x?
c. Interpret the slope and y-intercept in terms of the rulers.

Place an inch ruler and a centimeter ruler against each other so that they run in opposite directions.

23. Using the ruler arrangement you made as a source of (x, y) pairs, make a table like Alice's and Oliver's. Then make a graph.

24. Write an equation for the function that shows the relationship between the numbers in your table.

25. Interpret the slope and y-intercept in your equation and graph in terms of your rulers and their positions.