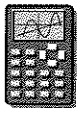
LESSON 10.6

Line Intersections

You will need:

graph paper

graphing calculator (optional)



POINTS ON LINES

1. On the same pair of axes, make accurate graphs of these three equations.

a. $3x + 5y = 9$ b. $6x + y = 18$

c. $4x + 2y = 30$

2. There is a point on each of the lines in problem 1 where the y-value is three times the x-value.

a. Find these points. Show your work.

b. The three points you found in part (a) should all lie on one straight line. What is the equation of this line?

3. Graph the line $4x + 2y = 6$. Then mark and label a point on the line for which

a. the y-coordinate is four times the x-coordinate;

b. y is twice x;

c. x is three less than y;

d. y is three less than x.

4. Add the graphs of the following lines to the axes you used in problem 3. Notice where each one intersects the line $4x + 2y = 6$.

a. $y = 4x$ b. $y = 2x$

c. $x = y - 3$ d. $y = x - 3$

5. Find the point on the line $2x - y = 6$ for which

a. the y-coordinate is one more than the x-coordinate;

b. the x-coordinate is 2/3 of the y-coordinate.

6. Explain the method you used to solve problem 5.

HOW MANY INTERSECTIONS?

7. Graph these three lines on the same pair of axes. Describe what you observe.

a. $x + 3y = 9$ $-x/3 + 9/3$

b. $2x + 6y = 18$

c. $x + 3y = 10$

8. Graph the line $2x - 3y = 4$. Then write an equation that has

a. the same graph;

b. a parallel graph.

For each pair of equations 9-12 tell whether the two graphs will be

a. the same graph;

b. parallel graphs;

c. intersecting graphs.

9. $2x + 9 = y$
$-4x - 18 = -2y$

10. $x - y = 7$
$x + y = 7$

11. $x + 6 = y$
$x + y = 6$

12. $x + y = 9$
$x + y = 7$

13. **Summary** Explain how to tell without graphing whether the equations of two lines have the same graph, parallel graphs, or intersecting graphs. Give examples.

HOW MANY SOLUTIONS?

Some pairs of equations 14-19 represent parallel lines. Some represent intersecting lines. Others represent the same line. *Without graphing*, find the point of intersection of each pair of lines, if it exists.

14. $2x - 3y = 7$
$3x - 4y = 15$

15. $x = 6 + 3y$
$3y = 3 + x$

16. $y - 12 = 4x$
$2y - 8x = 24$

17. $y = 42 - 4x$
$6x = 50 + 5y$

18. $y - 12 = 4x$
$2y = 8x + 24$

19. $2y - 2x = 7$
$y - x = 3.5$

20. **Summary** Explain, giving examples, and compare what happens when you try to solve the system if

a. the lines are parallel;

b. the equations represent the same line;

c. the lines meet in one point.

ADDING LINES

21. a. Graph the two lines on the same pair of axes.

$$3x + y = 7$$
$$-2x + y = -8$$

b. Label the point of intersection.

c. Add these two equations to get a third equation. Graph it on the same pair of axes. What do you notice?

22. a. Graph these two lines on the same pair of axes.

(A) $5x - 2y = 3$

(B) $2x + y = 3$

b. Label the point of intersection.

c. Get a third equation by adding.

(A) + (B) + (B)

Graph this equation on the same pair of axes. What do you notice?

23. Solve the system. $\begin{cases} 5x - 2y = 3 \\ 2x + y = 3 \end{cases}$

24. Here are two equations of lines.

$$2x + 3y = 5$$
$$x + 2y = 4$$

Use addition of these equations to get the equation of a *horizontal* line that passes through their intersection.

25. Solve the system. $\begin{cases} 2x + 3y = 5 \\ x + 2y = 4 \end{cases}$

26. **Summary** Explain how "adding lines" to get horizontal and vertical lines is related to solving systems of equations.

MORE MIND READING

27. Which of these problems has one solution? Which has an infinite number of solutions? Which has no solution? Explain.

a. I'm thinking of two numbers. Their sum is 10. Twice the first plus twice the second is 20.

b. I'm thinking of two numbers. Their sum is 6. Their difference is 10.

c. I'm thinking of two numbers. The second is 5 more than the first. The second minus the first is 6.

REVIEW ***SLOPE-INTERCEPT FORM***

28. The following questions are about the graph of $y = \frac{2}{3}x - 1$.

a. Where does it meet the y-axis?

b. If you move 2 units up and 3 units to the right from the y-intercept, where are you? Is that point on the graph? Explain.

c. If you move 2 units up and 3 units to the right from the point you found in part (b), where are you? Is that point on the graph? Explain.

d. Start anywhere on the line. Move 6 units up and m units to the right, to end up on the graph. What is m? Explain.

29. Describe a fast way to graph a line whose equation is given in slope-intercept form. Use an example.

30. Write these equations in slope-intercept form.

a. $3y = 4(2 - x)$

b. $4 - 3x = y - 2$

c. $y - 4 = 3(x - 2)$

d. $\frac{y}{2} = \frac{2 - 4x}{6}$