THINKING WRITING

7.A Cube Problems

THE PAINTED CUBE

Lea made a cube by gluing together 27 Lab Gear 1-blocks.

1. Make a sketch of what this cube would look like. What are its dimensions?

Lea painted the cube red on all six sides. Later, Mary and Martin were annoyed when they discovered what Lea had done. They needed the 27 one-blocks to do a hard factoring problem. Besides, they didn't think she should have been gluing and painting Lab Gear blocks.

2. a. When Mary and Martin broke Lea's cube apart into the 27 original small cubes, how many of the 1-blocks did they find to be painted red on three sides?
   b. How many were painted red on only one side?
   c. How many were painted red on two sides?
   d. How many had no red paint on them?
3. Repeat problem 2 for a 4-by-4-by-4 cube.
4. **Report** Write a report about problems 2 and 3. It should include, but need not be limited to, the following:
   - Show how you solved problems 2 and 3. Include sketches.
   - Look for patterns in your answers. Use them to guess the answers for a cube of side 5 and a cube of side 6. How can you check whether or not you are right?
   - Make a generalization to an n-by-n-by-n cube. Write expressions in terms of n for the number of cubes with 0 sides painted, 1 side painted, 2 sides painted, and 3 sides painted. (Explain why the four expressions should add up to n^3, and check that they do.)

CUBES IN CUBES

It is easy to see that there are 27 different 1-by-1-by-1 cubes in this 3-by-3-by-3 cube. It is harder to see how many different 2-by-2-by-2 cubes there are, because they overlap.

5. Figure out how many different 2-by-2-by-2 cubes there are in a 3-by-3-by-3 cube.
6. Think about a 4-by-4-by-4 cube. It contains how many
   a. 1-by-1-by-1 cubes?
   b. 2-by-2-by-2 cubes?
   c. 3-by-3-by-3 cubes?
   d. 4-by-4-by-4 cubes?
   e. cubes altogether?
7. Find how many cubes of each size there are in a 5-by-5-by-5 cube. Try to figure out a systematic way for counting the cubes.
8. **Report** Write a report about these cube problems. It should include, but not be limited to, the following:
   - Describe the strategy you used to answer problems 5, 6, and 7. Use sketches and explain your reasoning.
   - Make a generalization. In an n-by-n-by-n cube, how many cubes of each size (1-by-1-by-1, 2-by-2-by-2, 3-by-3-by-3, and so on) would there be? Write expressions in terms of n.
   - Test your generalization by trying it for a 7-by-7-by-7 cube. How many smaller cubes of each size should there be, according to your generalization? If you add all these numbers, do you get the correct total of 784? Show your work.