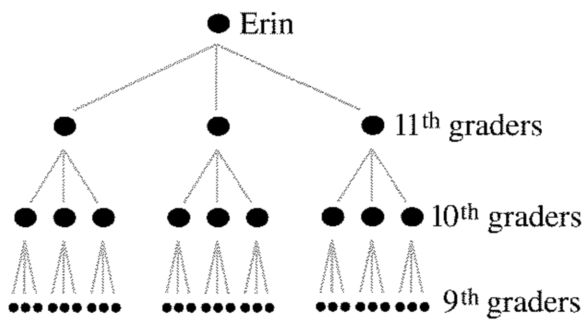
LESSON 7.8

Power Play

RAFFLE TICKETS

Erin is a senior at Alaberg High School and the director of the senior class play. To help pay for sets and costumes, she plans to raise money through a raffle. She is considering several plans for selling raffle tickets.

Erin's first idea was to have members of each class sell raffle tickets to the class below them. Erin would sell tickets to three 11th graders. Each of them would sell tickets to three 10th graders, who in turn would each sell tickets to three 9th graders, and so on. Erin started to draw a tree-diagram of her plan.



1. If Erin extended her plan all the way down to first grade, how many first graders would be buying tickets? Explain.

2. Make a table like the one following showing how many tickets would be bought by students in each grade. (The first entry in the table is based on the assumption that Erin bought one ticket for herself.) In the last column, express the number of tickets as a power of 3.

Grade	Tickets (number)	Tickets (as a power of 3)
12th	1	
11th	3	3^1
...		
1st		

3. Give several reasons why Erin's plan is not practical.

THE EXPONENT ZERO

The last column in your table above contained increasing powers of 3.

4. a. To follow the pattern, what should the exponent on the first power in the table be?
   b. Based on that pattern, what should 3^0 be equal to?

5. a. Copy and complete this table.

5^5	3125
5^4	
5^3	
5^2	
5^1	

   b. As you move down the columns, how can you get the next row from the previous one?
   c. Add another row to the bottom of the table. Explain how it fits the pattern.

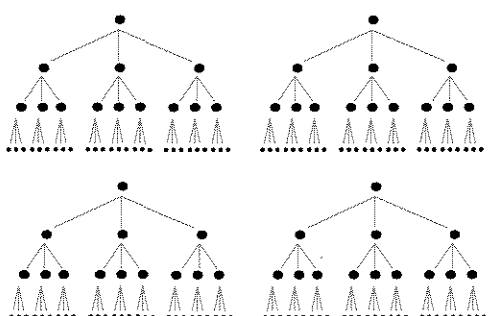
6. **Generalization** You have found the values of 3^0 and 5^0. Using patterns in the same way, find the values of 2^0 and 4^0. What generalization can you make?

7. **Summary** Many people think that a number raised to the zero power should be zero. Write a few sentences explaining why this is not true.

A BETTER PLAN

Erin needs a better scheme for selling raffle tickets. She decides to enlist the help of other seniors in the play. Each senior (including Erin) will buy a ticket for himself or herself, and sell a ticket to three juniors; each of the juniors will sell a ticket to three sophomores; and so on, down to the 8th grade.

Four more seniors help out:



8. Assume Erin gets five seniors to help (including herself).
   a. How many 8th graders would buy tickets?
   b. How is this number related to the number of 8th graders who would buy tickets if Erin does not get any other seniors to help?
   c. Express the answer to (a) as a number times a power of 3. Explain.

9. If Erin gets K seniors to help (including herself), how many 8th graders would buy tickets? Express the answer in terms of K.

10. Assume five seniors are involved, including Erin. As before, each student at every step buys one ticket, but now each student sells two tickets instead of three.
    a. How many 8th graders would buy tickets?
    b. Express the answer to (a) as a number times a power. Should you use a power of 2, a power of 3, or a power of 5? Explain your answer.

11. Assume K seniors are involved and each student sells M tickets.
    a. How many 8th graders would buy tickets? Express your answer in terms of K and M.
    b. How many Nth graders would buy tickets? Express your answer in terms of K, M, and N.

12. **Exploration** Erin hopes to sell 1500 tickets altogether. Find several values for K (the number of seniors) and M (the number of tickets sold per person) that make it possible to sell at least 1500 tickets, without going below 7th grade. For each plan, indicate the number of students who would be involved at each grade level. Which of those plans do you think is the most realistic?

REVIEW *WHICH IS GREATER?*

13. Which is greater?
a. $5 \cdot 3^{35}$ or $3 \cdot 5^{35}$
b. $5 \cdot 30^{35}$ or $30 \cdot 5^{35}$
c. $5 \cdot 300^{35}$ or $300 \cdot 5^{35}$

14. Which is greater?
a. $5^{35} \cdot 3^{35}$ or 15^{35}
b. 35^0 or 0^{35}

15. If a and b are each greater than 1, which is greater, $(ab)^{10}$ or ab^{10}? Explain.

REVIEW *A COMMUTATIVE LAW?*

Al announced, "I noticed that $4^2 = 2^4$ and $3^2 = 2^3$, so I generalized this using algebra to say $a^b = b^a$, always."

"That's a great discovery," said Beau. "This means that exponentiation is commutative!"

"Nice try, Al," said Cal. "It's true that 4^2 and 2^4 are both 16, but 3^2 is 9 and 2^3 is 8. They aren't equal."

Al was disappointed. "Round-off error," he muttered. "Close enough."

16. What did Beau mean when she said that exponentiation is commutative? Is she right or wrong? Explain, using examples to support your answer.

17. Is $4^2 = 2^4$ the only case where $a^b = b^a$? If it is, how can you be sure? If it isn't, how can you find others?

18. **Exploration** Which is greater, a^b or b^a? Of course, the answer to this question depends on the values of a and b. Experiment, and try to make some generalizations.

REVIEW/PREVIEW *CHUNKING*

19. Solve for y: $y^2 = 49$. (Remember there are two solutions.)

You can use the strategy of chunking to solve equations involving squares. For example, in problem 20, think of $(x + 3)$ as a chunk, and write two linear equations.

Solve.

20. $(x + 3)^2 = 49$

21. $(2p - 5)^2 = 49$

22. $(5 - 2p)^2 = 49$

23. $(6 + 2r)^2 = 49$