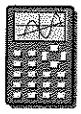
LESSON 7.5

# Remarkable Identities

**You will need:**

- the Lab Gear 
- graph paper 
- graphing calculator (optional) 

## REVIEW MULTIPLYING PATTERNS

**1.** Find these products.

a. $(y - 3)(y + 3)$

b. $(y + 5)(y - 5)$

**2.** What is the pattern in problem 1?

**3.** Does the pattern still hold for $(2x - 1)(2x + 1)$? Explain.

**4.** Find these squares.

a. $(y - 3)^2$ b. $(y - 5)^2$

c. $(y + 3)^2$ d. $(y + 5)^2$

**5.** What is the pattern in problem 4?

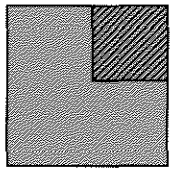
**6.** Does the pattern still hold for $(2x - 1)^2$ and $(2x + 1)^2$? Explain.

## THREE IDENTITIES

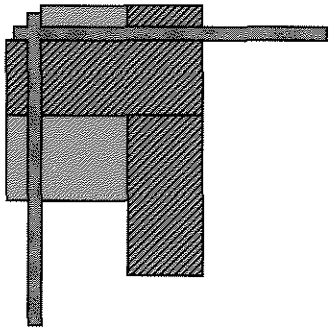
**7.** True or False? The square of a sum is equal to the sum of the squares. Explain, using a sketch.

**8.** Describe a shortcut for finding the square of a sum.

The expression $y^2 - x^2$ is the difference of squares. (Remember that shaded blocks are *upstairs.*)



The expression $(y - x)^2$ is the square of a difference.



**9.** True or False? The square of a difference is equal to the difference of the squares. Explain.

**10.** Describe a shortcut for finding the square of a difference.

**11.** Find the products.

a. $(y + x)^2$

b. $(y - x)^2$

c. $(y - x)(y + x)$

As you know, identities are algebraic statements that are always true. The three that are shown in problem 11 are especially important and useful. You should memorize them. For example, using the second one,

$$(2x - 5)^2 = (2x)^2 - 2(2x)(5) + 5^2$$
$$= 4x^2 - 20x + 25.$$

**12.** Multiply by using one of the identities. You may check your answers with the Lab Gear or by setting up the multiplication as a table.

a. $(3x - 2)^2$

b. $(3x + 2)^2$

c. $(3x - 2)(3x + 2)$

Even if you don't use the identities for multiplying, it is useful to memorize them in order to recognize them quickly when trying to factor a trinomial. Knowing them is also useful for understanding the solution of quadratic equations.

## A CASE OF MISTAKEN IDENTITY

Some of the most common mistakes of math students concern the identities you have learned about in this chapter. Even after having learned the identities, students often forget and write $(x + y)^2 = x^2 + y^2$ or $(x - y)^2 = x^2 - y^2$. This mistake causes math teachers to tear their hair in desperation.

**13.** **Report** Write an article or create a poster that you think would help other students avoid these mistakes. (Math teachers all over the world would greatly appreciate your help.) Include explanations, sketches, and examples. Make your article or poster appealing, eye-catching, or humorous so that other students will want to read it.

## FACTORING

**14.** Factor these trinomials.

a. $9x^2 + 6x + 1$

b. $x^2 - 6xy + 9y^2$

c. $4x^2 + 4xy + y^2$

d. $9x^2 - 25$

e. $4x^2 - 4y^2$

f. $a^2x^2 + 2acx + c^2$

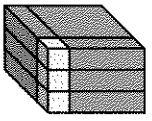
**15.** Use the Lab Gear to make as many different rectangles as you can with $3x^2 + 12x + 12$. Write a product corresponding to each rectangle.

**16.** The figure below shows a box with a square base.

a. Write an expression for the volume of the box in the form

*Height* · *Area of Base.*

b. Write an expression for the volume of the box in the form *Height* · $(Side)^2$.



**17.** Each of these expressions gives the volume of a box that has a square base. For each one, write an expression of the form *Height* · $(Side)^2$. You may want to use the Lab Gear.

a. $3x^2 + 12x + 12$

b. $8x^2 + 8x + 2$

c. $3x^2 + 6xy + 3y^2$

d. $2y^2 + 12y + 18$

e. $xy^2 + 2xy + x$

**18.** Each of these polynomials gives the volume of a box that has a square base. For each one, write an expression of the form *Height* · $(Side)^2$, without using the blocks. (Hint: The height of the blocks is the factor that is common to all three terms.)

a. $27x^2 + 54x + 27$

b. $60y^2 + 60y + 15$

c. $50x^2 + 100xy + 50y^2$

d. $16y^2 + 96y + 144$

e. $6x^2y + 24xy + 24y$

## SQUARING TRINOMIALS

Do you think there is a pattern for the square of trinomials? Experiment with these problems.

**19.** $(x + y + 2)^2$

**20.** $(x + y - 5)^2$

**21.** Describe the pattern you discovered in problems 19 and 20.

**22.** What is $(a - b + c)^2$ equal to? Use the pattern you discovered, then check your answer by using the distributive law very carefully.

## CUBES OF SUMS

**23.** Find an identity for the cube of a sum. Lab Gear models using 3-D blocks may help. Explain why the cube of a sum is not the sum of the cubes.

## PUZZLE SUM OF SQUARES

**24.** $5x^2 + 20x + 25$

Think of the Lab Gear blocks representing this polynomial. The polynomial is not a perfect square, so you cannot rearrange it into a single square. However, it can be arranged into a *sum of squares*. Figure out how you would do it.

## REVIEW/PREVIEW ALWAYS, SOMETIMES, OR NEVER TRUE?

**25.** On the same axes, graph $y = 12 - x$ and $y = 8 - x$.

**26.** Always, sometimes, or never true? (Explain your reasoning in each case.)

a. $12 - x > 8 - x$

b. $12 - x > 13$

c. $8 - x > 12 - x$

d. $4 > 8 - x$

e. $-4 > 8 - x$

**27.** Always, sometimes, or never true? (If sometimes true, give the values of $x$ that make it true.)

a. $x > 2x - 8$

b. $2x - 5 > 2x - 8$

c. $x < 2x - 5$