LESSON 6.9

Rational Expressions

You will need:

the Lab Gear 

COMPARING RATIONAL EXPRESSIONS

To add, subtract, multiply, and divide fractions involving variables, use the same rules you use for numerical fractions.

1. Review the rules for adding, subtracting, multiplying, and dividing fractions, using an example of each kind.

2. For each expression, substitute 1, 2, and 9 for x and perform the indicated operation. In which problem is the answer the same, regardless of the value of x?

a. $\frac{5}{x} \cdot \frac{x}{5}$ b. $\left(\frac{5}{x}\right) / \left(\frac{x}{5}\right)$

c. $\frac{5}{x} + \frac{x}{5}$ d. $\frac{5}{x} - \frac{x}{5}$

A *rational number* is any number that can be written as a ratio of integers. A *rational expression* is an expression that involves a *ratio.* A very simple rational expression is the rational number 1/2, which is the ratio of 1 to 2. A more complicated rational expression is $(x^2 + 3x + 4)/(x^3 - 99)$, which is the ratio of two polynomials.

3. With the numbers 3 and 4, you can write the ratio 3/4 or the ratio 4/3.

a. Which is greater, 3/4 or 4/3?

b. Which is greater, 3/4 of 4/3 or 4/3 of 3/4? Explain.

4. For each pair of expressions below, write:

A if the expression in the first column is greater

B if the expression in the second column is greater

? if the value of x determines which one is greater

Explain your answers.

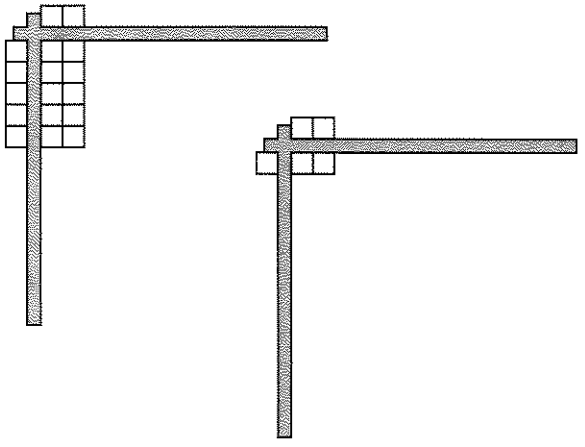
a. $\frac{x}{5}$ $\frac{x-2}{5}$

b. $x - \frac{2}{5}$ $\frac{x-2}{5}$

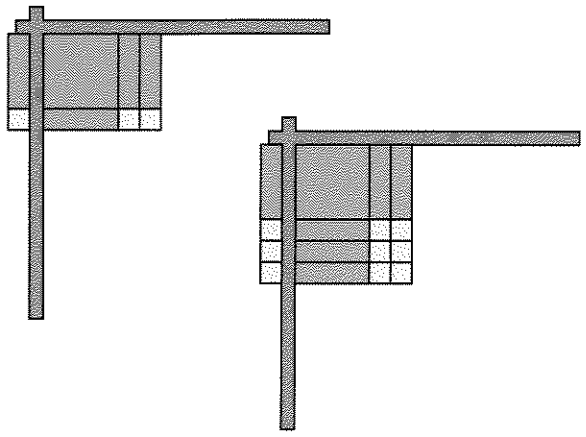
c. $\frac{5}{x}$ $\frac{5}{x-2}$

EQUIVALENT RATIONAL EXPRESSIONS

By dividing, you can show that two fractions represent the same ratio. For example, as the figure shows, 10/5 equals 2/1.



The same thing sometimes works with polynomials. As shown in the figure, the rational expression $(x^2 + 3x + 2)/(x + 1)$ is equal to $(x^2 + 5x + 6)/(x + 3)$ because the result of both divisions is the same.



5. What is the result of both divisions?

(**Note:** These two rational expressions are **not** equal when $x = -1$ or $x = -3$. Can you see why? Try substituting these numbers for x and see what happens.)

6. Are these rational expressions equal? Explain. You may use a sketch.

$(xy + 2x)/x$ $\qquad$ $(y^2 + 2y)/y$

7. For each problem, find a number or expression you could put in the box that would make the two rational expressions equal. Explain each part, perhaps using a sketch.

a. $\frac{3x}{x}$ $\qquad$ $\frac{3x + 6}{\square}$

b. $\frac{18}{6}$ $\qquad$ $\frac{15}{\square}$

c. $\frac{\square}{x + y}$ $\qquad$ $\frac{2x + 2y}{2}$

d. $\frac{x^2 + 8x + 12}{\square}$ $\qquad$ $\frac{2x + 12}{2}$

SOLVING EQUATIONS INVOLVING RATIOS

8. **Exploration** The equation

$$\frac{x - 3}{5} = x + 1$$

cannot easily be modeled with the Lab Gear. Try to solve it using any technique you have learned. Compare your method and your answers with other students' work.

Lea and Earl both tried to solve this equation. They got different answers.

Lea's work	**Earl's work**
$5\left(\frac{x - 3}{5}\right) = (x + 1) \cdot 5$	$5\left(\frac{x - 3}{5}\right) = (x + 1) \cdot 5$
$x - 3 = 5x + 5$	$5(x - 3) = 5(x + 1)$
$-3 = 4x + 5$	$\frac{1}{5} \cdot 5(x - 3) = \frac{1}{5} \cdot 5(x + 1)$
$-2 = 4x$	$x - 3 = x + 1$
$-0.5 = x$	impossible

9. Who is right, or are they both wrong? Copy each student's work and write an explanation beside each step telling what was done. If a step is incorrect, explain why and make a correction.

Solve these equations. Show your work and write a brief explanation of each step.

10. $\frac{6 - x}{8} = \frac{x}{2}$ $\qquad$ **11.** $\frac{6 - x}{8} = \frac{x}{5}$

12. $\frac{x + 5}{x + 7} = 3$ $\qquad$ **13.** $\frac{6 + 2x}{x + 7} = \frac{4}{5}$

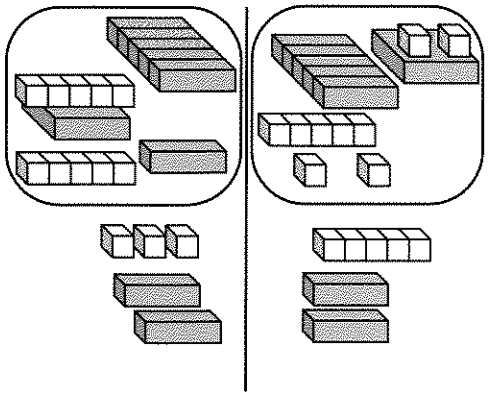
14. $\frac{x - 3}{2} = x + 5$

15. **Summary** Describe a general method for solving equations like those in problems 10-14. Include the solution of an example you made up.

REVIEW EQUATION SOLVING

Write the equation shown by the blocks, then solve it. If you use the Lab Gear, write equations to show some of the steps as you move your blocks. If you don't, show all your work.

16.



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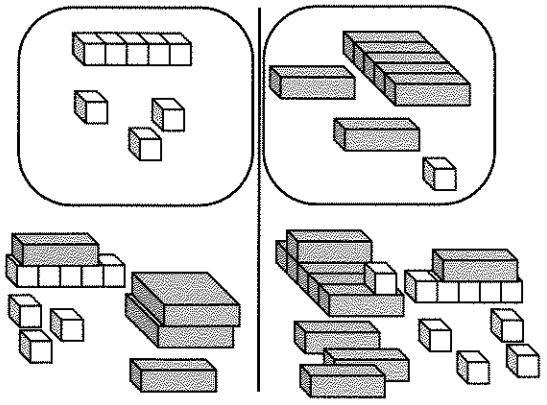

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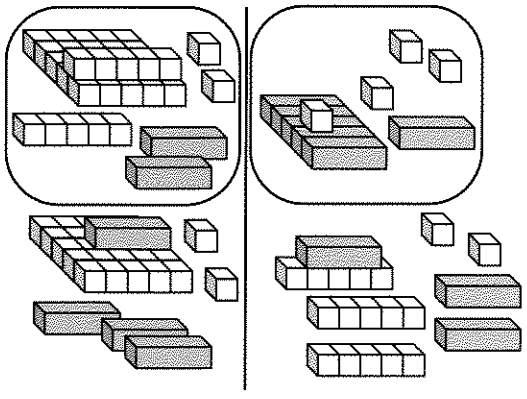

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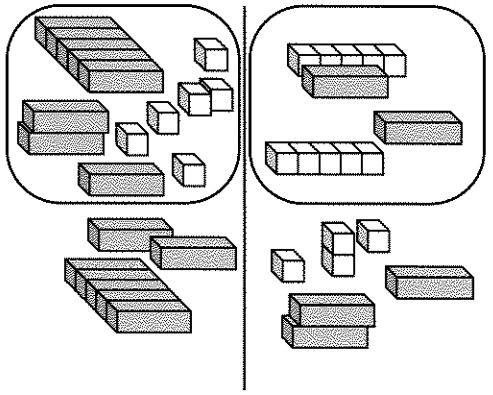

Solve.

20. $(x + 3)^2 = (x - 3)(x + 4)$

21. $(x - 1)^2 = (x + 2)(x - 6)$

PUZZLES ORDER OF OPERATIONS

Keeping in mind order of operations, insert as many pairs of parentheses as needed, to make these equations true.

22. $4 \cdot 2 + 3 = 20$

23. $\frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

24. $5 \cdot 3 - 2 + 6 = 35$

25. $3^2 + 2 \cdot 7 - 4 = 33$

26. $\frac{1}{3} \cdot 6 + 4 \cdot \frac{2}{6} + \frac{1}{3} = \frac{7}{3}$

27. $1 - 2 \cdot 2 + 5 \cdot 6 = -42$