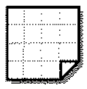
LESSON 6.8

Solving Techniques: Multiplication and Division

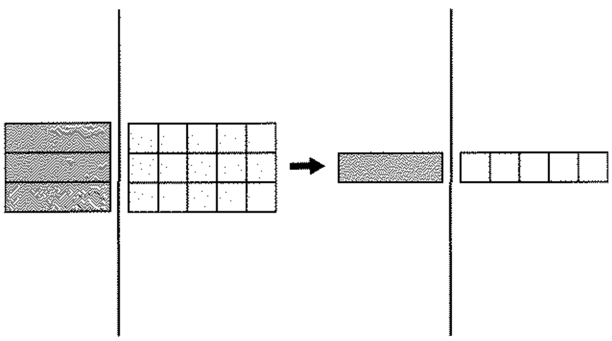
You will need:

the Lab Gear 

graph paper 

Another key to solving equations is the fact that you can *multiply* or *divide both sides by the same number* (as long as it's not zero).

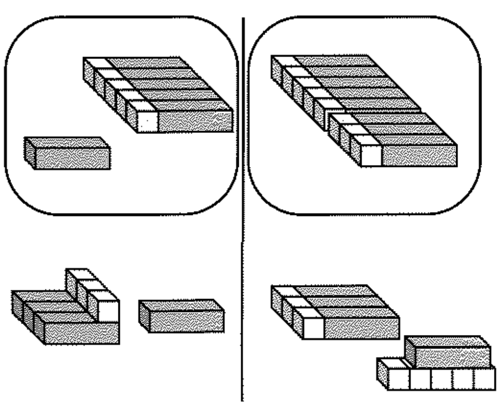
For example, if $3x = 15$, then divide both sides by 3, and you find that $x = 5$.

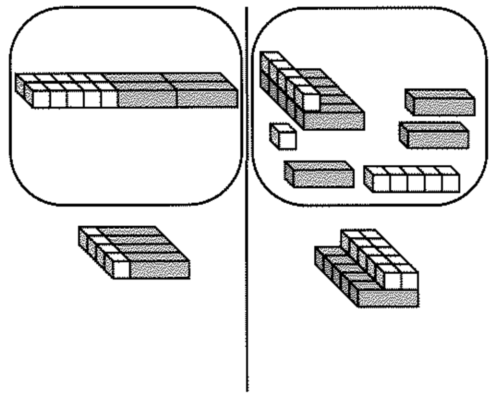


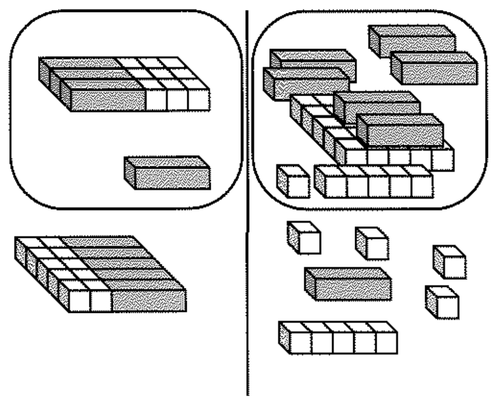
Of course, some divisions cannot be shown easily with the blocks. If you end up with $4y = 7$, then dividing both sides by 4 will reveal that $y = 7/4$. This is impossible to show with the Lab Gear.

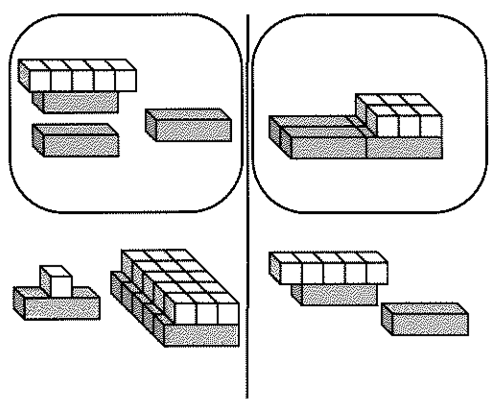
USING THE LAB GEAR

Write and solve these equations.

1. 

2. 

3. 

4. 

USING RECIPROCALS

Solve these without the Lab Gear.

5. $\frac{2}{3}x = 18$ **6.** $\frac{1}{5}x = 99$

Solve these by multiplying or dividing first, and then again by first distributing the number in front of the parentheses. You should get the same answers by both methods.

7. $7(x - 2) = 30$ **8.** $12(x + 6) = 48$

9. $\frac{1}{3}(2x - 4) = 5$ **10.** $\frac{4}{5}(2 - 8x) = 16$

11. $\frac{1}{2}(2x - 4) = 5$ **12.** $\frac{5}{4}(2 - 8x) = 16$

13. **Summary** Use examples.

a. Explain how to decide which of the two methods (distributing first or later) one should use in problems 7-12.

b. Explain how to decide what number to multiply or divide both sides by when solving an equation.

14. Start with $x = -3$.

a. Create an equation by adding and/or subtracting the same amount from both sides repeatedly, and by multiplying and/or dividing both sides by the same amount repeatedly. Write the final equation on paper and give it to a classmate.

b. Solve a classmate's equation. If you both do your work correctly, the solution should be -3.

SOLVING FOR *y*

You have learned to multiply or divide by a number when solving an equation containing one variable. This is also a useful technique when working with equations containing two variables, such as this one, $4y - 8x = 0$.

In a two-variable equation, it is often useful to solve for one variable *in terms of* another. This means that one variable is alone on one side of the equation.

By adding $8x$ to both sides, it is easy to rewrite this equation so that the y's are on one side and the x's are on the other:

$$4y = 8x.$$

Dividing both sides by 4 gives

$$y = 2x.$$

Transform each equation below so that y is in terms of x. You may use the Lab Gear.

15. $3y - 6x = 9$ **16.** $6x - 3y = 12$

17. $x - y = 1$ **18.** $6x - 5y = 0$

EQUIVALENT EQUATIONS

19. Draw axes and plot three (x, y) pairs that satisfy the graph of $4y - 8x = 0$. Describe the graph.

20. Find three (x, y) pairs that satisfy $y = 2x$ and draw the graph. Compare it with the graph in problem 19. What do you notice? Explain.

If equations in two variables have the same graph on the Cartesian coordinate system, they are called *equivalent equations.*

21. Explain how you could have determined *without graphing* that the equation $4y - 8x = 0$ is equivalent to $y = 2x$.

22. a. Write an equation that is equivalent to $6y = 12x$, but looks different.
    b. Describe what the graphs of both equations would look like.

For each group of equations decide which ones, if any, are equivalent equations. If you are unsure, you might want to solve the equations for y, make some tables, or draw some graphs.

23. $x + y = 2$
    $2x + 2y = 2$
    $2x + 2y = 4$

24. $x/y = 12$
    $y/x = 12$
    $y = 12x$

25. $3x - y = 6$
    $2y = 6x - 12$
    $y - 3x = 6$

26. $0.8x = y$
    $x - 0.2x = y$
    $y - 4/5x = 0$

27. $1.2\,x = y$
    $x + 0.2x - y = 0$
    $2.4x - 2y - x = 0$

PUZZLES **AGE RIDDLES**

28. At age 3, Henry could count to 12. How far could he count by age 21?

29. Augustus De Morgan lived in the nineteenth century. He said, "I was x years old in the year x^2." In what year was he born?

30. Diophantus spent one-sixth of his life in childhood and one-twelfth of his life in youth. He spent one-seventh more of his life as a bachelor. Five years after he was married, his son was born. His son lived half as long as his father and died four years before his father. How many years did Diophantus live? How old was he when he got married?

31. Make up an age riddle.

32. Solve a classmate's riddle.

RESEARCH **FAMOUS MATHEMATICIANS**

33. Prepare a report about Diophantus or Augustus De Morgan. What were their contributions to mathematics?