LESSON 5.1

Constant Sums

You will need:

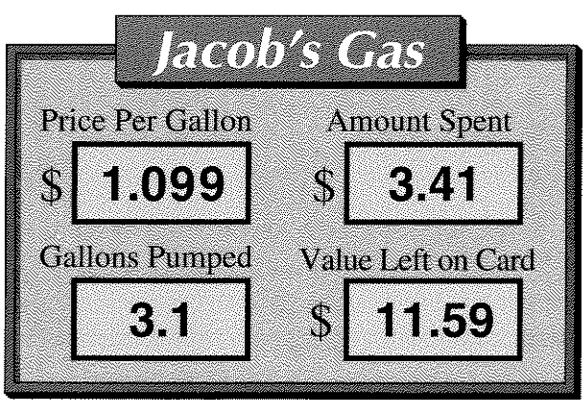
graph paper

AT THE GAS STATION

When Oliver and Alice pulled up to the self-serve island at Jacob's gas station, they noticed a new sign:

Buy Gas Card in Office

They went into the office, which was decorated with photographs and cartoons. The attendant Harold explained to them that he could sell them a gas card for any amount from 5 to 100 dollars. They would put it in the special slot in the pump, and pump gas as usual. The value of the card would automatically go down, and a display on the pump would indicate the value left in the card. After getting gas, there would be no need to go back to the office, unless they wanted to trade the card back for cash. (This could be done only if the card had less than $5 left on it.) Or they could use the remaining money left on the card the next time they stopped at Jacob's.



1. Look at the dials in the figure. How much did Oliver and Alice pay for their gas card?

2. Oliver and Alice plan to buy about $10.00 worth of gas. List at least five other pairs of numbers that will appear on the last two dials while they are pumping gas.

3. When exactly 11 gallons have been pumped, what numbers will appear on the four dials?

4. **Generalization** When D dollars have been spent, what is the value left on the card?

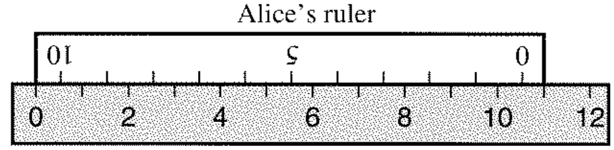
5. When G gallons have been pumped,

a. how many dollars have been spent?

b. what is the value left on the card?

FUNCTION DIAGRAMS FROM RULERS

Alice wanted to know how long her ruler was. Oliver suggested she measure it with a longer ruler, as in this figure.



6. How long is her ruler?

Oliver had to write about function diagrams for algebra. (His class was using this textbook, and in a curious coincidence, they were doing exactly this page!) He decided to use the rulers as a way to get tables of x- and y-values and build a function diagram from them. He used the rulers setup to create a table that started this way.

x	y
1	9.5
2	8.5
3	7.5

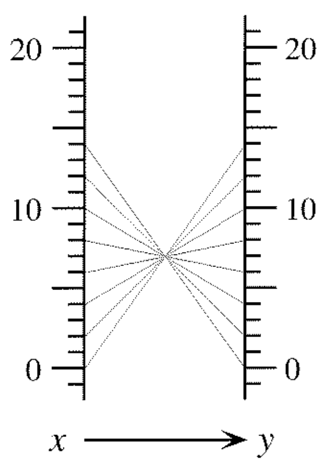
7. Describe the pattern for the numbers in the table. Does it matter which ruler you use for x and which for y? Explain.

8. Write a function of the type $y =$ *an expression in terms of* x for Oliver's table.

9. Make a function diagram for Oliver's table. (Use at least five in-out lines.)

10. Use rulers to create two more tables, and for each, write a function and make a function diagram. At least one of them should match 0 with a number other than a whole number.

11. How could you set up rulers to get this function diagram? Explain.



All the function diagrams you just drew have something in common. For each one, the sum of all the (x, y) pairs is a constant. We could call them *constant sum* functions.

12. Summary Write an illustrated summary describing what you noticed about diagrams of constant sum functions. It should include, but not be limited to, examples and answers to the following questions:

- Do the in-out lines meet in one point?
- If they do, could you predict the position of this point if you knew the value of the constant sum?

GRAPHS OF CONSTANT SUMS

13. a. On a pair of axes, plot these (x, y) pairs.
(2, 4) (4, 2) (-1, 7) (8, -2)

b. In words, we could describe the pattern of the (x, y) pairs by saying that the sum of x and y is always six. How would you write this using algebra?

c. Find three more (x, y) pairs that fit this pattern, and add the points to your graph.

d. Connect all the points with a line or curve. Describe the graph.

14. a. Find points such that $x + y < 6$. Where are they in relation to the graph in problem 13?

b. Repeat for $x + y = 6$.

c. Repeat for $x + y > 6$.

15. Find a point (x, y) such that $x = y$ and $x + y = 6$. Label it on the graph.

16. Choose a positive value for S and make a table of (x, y) pairs that satisfy the equation $x + y = S$. Use your table to make a graph.

17. Experiment with some other constant sum graphs. Try several different positive values for S. For each one, make a table of at least five (x, y) pairs having the sum S. Then draw a graph. Draw all your graphs on the same pair of axes.

18. Do any of the lines go through the origin? If not, do you think you could pick a number for your sum so that the line would go through the origin? Explain.

19. Repeat your investigations for equations of the form $x + y = S$, where S is negative. Keep a record of what you try, using tables and graphs.

20. Report Write an illustrated report summarizing your findings about constant sum graphs. Your report should include neatly labeled graphs with accompanying explanations. Include answers to the following questions:

- Were the graphs straight lines or curved, or were there some of each?
- Without drawing the graph, could you now predict which quadrants the graph would be in, if you knew the value of S? Explain.
- Without drawing the graph, could you predict the x-intercepts and y-intercepts of the graph, if you knew the value of S? Explain.
- What determines whether the graph slopes up or down as it goes from left to right? Could you predict this without graphing if you knew the value of S? Explain.
- Do any of your graphs intersect each other? If so, which ones? If not, why not?