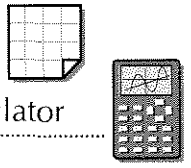
LESSON 4.7

Real Numbers and Estimation

You will need:

graph paper

graphing calculator (optional)



MEASUREMENT ERROR

The three tables in Lesson 6 contained data that were invented. You can tell because all the points lie exactly on a line. In real experiments measurements can never be exact. This table contains more realistic data.

Volume	Weight
10 ml	32 g
20 ml	63 g
50 ml	146 g
80 ml	245 g

1. Draw and label a pair of axes and plot these points.
2. You cannot draw a straight line through all the points, but draw one that passes as closely as possible to all of them. Be sure your line goes through the origin. (Explain why it must pass through the origin.)
3. What is the equation of the line you drew? (Hint: Choose a point on the line to help you figure this out.)
4. Based on your answer to problem 3, what would you estimate the density of the substance to be?
5. Find the ratio of weight to volume for each data point in the table.
6. Based on your calculations in problem 5, what do you estimate the density of the substance to be?
7. **Summary** You estimated the density of this substance in two different ways. If you did not get the same answer using both methods, explain any differences. Which method do you like better, and why?

ESTIMATING TEMPERATURE

In Chapter 3, Lesson 8, you learned this rule for converting Celsius to Fahrenheit:
Multiply the Celsius temperature by 1.8. Add 32 to the result.
If

F = the Fahrenheit temperature and
C = the Celsius temperature,

then this statement can be written as a function:

$$F = 1.8\,C + 32.$$

8. Draw and label a pair of axes with F on the y-axis and C on the x-axis. Make a table of values, using values of C from -10 to 30. Use your table to graph the function $F = 1.8\,C + 32$. Label a few points on your graph.

Abe doesn't like to multiply by 1.8. Since 1.8 is a little less than 2, and 32 is a little more than 30, he made up this rule for estimating: *To estimate the Fahrenheit temperature, multiply the Celsius temperature by 2 and add 30.*

9. Using the letters C and F as was done in problem 8, write a function for Abe's rule.
10. Make a table using values of C from -10 to 30 for the function you wrote for Abe's rule. Use your table to graph the function on the same pair of axes as you used in problem 8.

11. Compare the two graphs.

a. How far off would Abe's estimate be if the Celsius temperature were 0?

b. Compare the result from Abe's estimation method with the exact values for several other temperatures. Be sure to try some negative Celsius temperatures. Do you think Abe's method is a good one? Why or why not?

c. There is one temperature for which Abe's estimation method gives the exact value. What is it?

12. For what range of temperatures would you judge Abe's method to be acceptable? Explain.

Sometimes exact answers are important. In everyday life, estimates or *rules of thumb* are often just as good. For example, Mr. and Mrs. Gral, who are planning a trip to Europe, are not really interested in knowing how to make exact temperature conversions. They just want some advice about what to wear.

13. Bea and Abe are making a chart for their parents' reference. Complete it.

Celsius temperature between __ and __	**You should wear:**
	your coolest clothes
	a sweater
	a coat
	heavy coat, gloves, hat, and scarf
	a space suit

ESTIMATING A TIP

Here is a method to figure out how much tip to leave for the server at a restaurant. Say the bill was for $20.73.

- Round up to the next even whole number of dollars, in this case 22.
- Add half of the number you got to the number, in this case 22 + 11 = 33.
- Round up to the next multiple of five, in this case 35.
- Divide by ten to get the tip, in this case $3.50.

14. What percentage of $20.73 is $3.50? (Round off your answer.)

15. Does this method always give the same percentage of the bill? Try it for several amounts to see whether the percentage varies. If it does, what seem to be the lowest and the highest value it will give?

Here is another method to figure out the tip.

- Divide the amount of the bill by ten. (In this case you would get $2.07.)
- Multiply the result by two. (In this case you would get $4.14.)
- Take the average of the two numbers, rounded to the nearest nickel.

16. a. What is the tip by this calculation?

b. What percentage of the bill is it?

17. Does the second method always give the same percentage of the bill? Explain.

18. Summary Compare the two methods. Explain which one you prefer and why.

19. What percentage of the bill do you think is an appropriate tip? Create your own method to figure it without a calculator.