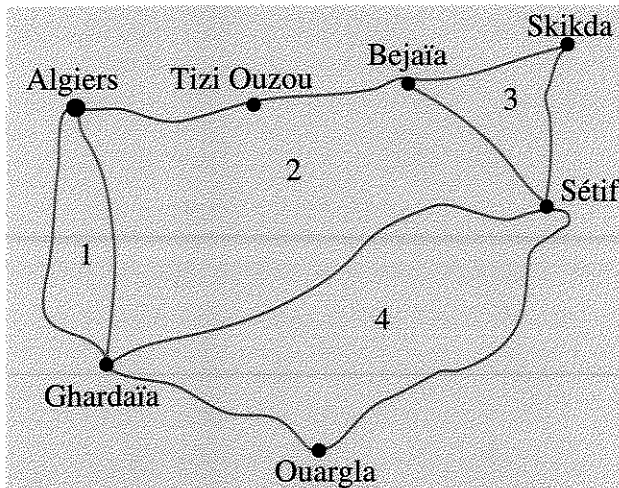# 2.C Towns, Roads, and Zones

This is a simplified road map of part of Algeria. It shows 7 towns and 10 roads. For the purposes of this lesson we will call any area completely surrounded by roads, (and not crossed by any road,) a *zone*. As you can see, there are 4 zones on this map.

Rules: Each town is connected to all the others by roads (not necessarily a direct connection); all roads begin and end at a town. It is possible for a road to connect a town to itself. It is possible for more than one road to connect two towns.

In maps like this one there is a relationship between the number of towns, roads, and zones. Your goal in this lesson is to find it. The relationship was discovered by the Swiss mathematician and astronomer Leonhard Euler. It is part of a branch of geometry called *topology*, which he created.
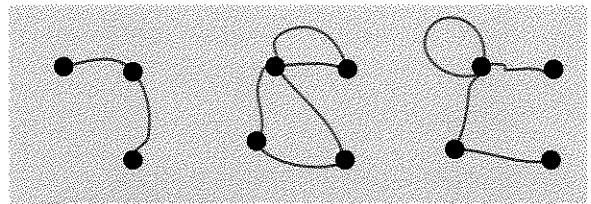
1. **Exploration** Make many different "maps" like the ones above. Keep track of the number of roads, towns, and zones in a table. Try to find a pattern in the relationship of the three numbers. (If you cannot find a relationship between all three numbers, keep one of the numbers constant and look for a relationship between the other two.)

2. Make at least six different three-town maps. What is the relationship between number of roads and the number of zones? Express it in words, and write r (the number of roads) as a function of z (the number of zones).

3. Make at least six different four-town maps. What is the relationship between number of roads and the number of zones? Express it in words and write a function.

4. Make at least six different five-road maps. What is the relationship between the number of towns and the number of zones? Express it in words and write a function.

5. Make at least six different six-road maps. What is the relationship between the number of towns and the number of zones? Express it in words and write a function.

6. Make at least six different four-zone maps. What is the relationship between the number of roads and the number of towns? Express it in words and write a function.

7. **Report** Write an illustrated report describing what you have learned about towns, roads, and zones. Give examples. Your report should answer the following questions, but not be limited to them:

- If there are t towns and r roads, how many zones are there?
- If there are t towns and z zones, how many roads are there?
- If there are r roads and z zones, how many towns are there?

8. **Project** Euler

Find out about Leonhard Euler and/or the Koenigsberg Bridge Problem. Prepare an oral presentation or a bulletin board display.