

DOING DISHES

Abe agreed to do the dishes daily in exchange for one cent on April 1st, two cents on April 2nd, four cents on April 3rd, and so on, doubling the amount every day.

- To find out how much money Abe was earning, make a table like this one, for at least the first ten days.

Day #	Cents	Total
1	1	1
2	2	3
3	4	7
4

- How are the numbers in the *Cents* column calculated?
- How much money did Abe get paid on April 30? Explain how you figured out the answer. Do you think you could talk your parents into an arrangement like this?
- Study the table, looking for a pattern in the *Total* column. Describe the pattern.
 - How much money did Abe make altogether during the month of April?

Definitions: Exponents

Exponentiation, or *raising to a power*, is the operation of multiplying a number by itself repeatedly. The number that is multiplied is called the *base*. The number of factors is called the *exponent*.

Examples:

- The expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is written 2^5 , where 2 is the base and 5 is the exponent.

- You are already familiar with squaring and cubing, which are special cases of exponentiation in the case of raising to the second and third powers.
- The numbers in the Cents column in the above table are called the *powers of 2*, because they can be obtained by raising 2 to different powers.

Notation:

- On calculators, it is not possible to use this notation. Instead, 2^5 is entered as $2 \square \overset{y^x}{\square} 5$, $2 \square \overset{x^y}{\square} 5$, or $2 \square \overset{\wedge}{\square} 5$.
- On computers, most word processors allow the user to type exponents (called *superscripts*).
- Computer programming languages use 2^5 , $2**5$, or `POWER 2 5`.

5. Generalization

- How much money did Abe make on the n^{th} day of April? (Watch out.)
- What is the number in the *Total* column on day n ? Explain.

EXPONENTIAL NOTATION

The number 64 can be written in exponential notation as 2^6 or 8^2 . (Check this with your calculator or by mental multiplication.)

- Find another way to write 64 in exponential notation.
- Write each of these numbers in exponential notation. Do not use 1 as an exponent. If possible, find more than one way. It may help to use your calculator.

a. 81	b. 1
c. 1024	d. 625
e. 6561	f. -512

CHAIN LETTER

Lara received this letter.

Dear Lara,

Send copies of this letter to five people, or the most terrible bad luck will afflict you. One man broke the chain, and a flower-pot fell on his head, giving him a terrible headache which continues to this day.

Don't look a gift-horse in the eye.
Rome was not built in a pond.
Don't cry over spilt tears.

Please do not break the chain!
It was started in 1919 by a psychic.

Bea

Assume that the chain is not broken, and that each person who receives it takes a week to send out five copies.

8. After one week, five people receive Lara's letter. After another week, how many people receive the letter? Make a table like the following for the first ten weeks.

Week #	Letters received this week	Total number received so far
1	5	5
2	25	30
3

9. How many weeks until the number of letters received that week is greater than the population of the United States?
10. How many letters were received in the n^{th} week?
11. If each person made six copies of the letter instead of five, how would your answer to problem 10 change?
12. Do you think that the chain was started in 1919? Explain why or why not.
13. How do the assumptions we made to solve this problem compare with what happens in the real world with chain letters?

GETTING HELP

Assume Lara gave a copy of the letter to Lea and they each sent five copies in the first week.

14. If everything continues as in the previous section, how many people receive the letter? Make a table like the following for the first five weeks.

Week #	Letters received this week	Total number received so far
1	10	10
2	50	60
3

15. Write the number of letters received in the 10^{th} week as an expression *using exponents*.
16. How many letters were received in the n^{th} week?
17. If each person asked a friend to help in the same way, how would your answers to problems 14-16 change?