LESSON 1.11

Word Figures

WORD TRIANGLES

Imagine you have a supply of letter tiles, and you use them to make word triangles like this one.

A
L A
A L E
R E A L

Rules: Each row contains the letters of the previous one, plus one more. It's OK to scramble the letters from one row to the next.

1. Extend this word triangle.

2. Make a word triangle with your own letters.

3. How many letter tiles are used in a five-row word triangle?

4. Make a table like this, extending it to ten rows.

Rows	Tiles
1	1
2	3
3	6
4	...

5. The numbers you found in problem 4 (1, 3, 6, ...) are called the *triangular numbers*. Explain how they are calculated.

6. Extend the above word triangle up to **ARGUABLE**. (Along the way, you might use **ALGEBRA**.)

WORD LADDERS

Rules: From one row to the next, change one letter only. It's OK to scramble the letters.

For example:

R E A L
L E A D
L O A D
F O L D
F O O D

7. Make up a word ladder with your own letters. Choose your word length, and use as many rows as you need. It's fun to choose related words for the beginning and end of the ladder, like **CAR** and **BUS**.

The above example, from **REAL** to **FOOD**, took four steps (and five rows). It is an example of a *perfect* word ladder. For a word ladder to be called perfect, two things must be true:

a. Every letter from the original word must be changed in the final word.

b. If the word has *n* letters, the ladder must take exactly *n* steps.

For a five-letter word, a perfect ladder would take five steps (one per letter) and therefore six rows.

8. How many tiles would a five-letter perfect word ladder require?

9. Make a table of the number of tiles required for perfect word ladders, extended to word length 10.

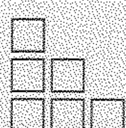
Word Length	Tiles
1	2
2	6
3	12
4	...

10. The numbers you found in problem 9 (1, 6, 12, ...) are called the *rectangular numbers*. Explain how they can be calculated.

11. Make up a word ladder from **MATH** to **GAME**.

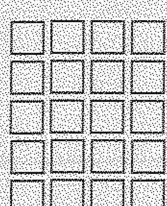
PUTTING IT TOGETHER

12. This figure shows the third triangular number.



Draw a sketch of two copies of this triangle, arranged together to make a rectangle.

13. This figure shows the fourth rectangular number.



Show how you could divide it into two equal triangular numbers.

14. **Summary** Describe the relationship between triangular numbers and rectangular numbers.

15.

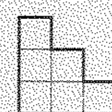
a. Explain how to calculate triangular numbers by first calculating rectangular numbers.

b. Calculate the 100th triangular number.

POLYOMINO PUZZLES

16. Using graph paper and scissors, or interlocking cubes, make a set of polyominoes having area greater than 1 and less than 5. You should have one domino, two trominoes, and five tetrominoes, for a total of eight puzzle pieces with no duplicates.

17. Using the same unit as you used for the puzzle pieces, draw staircases with base 3, 4, 5, 6, and 7. The first one is shown here.



Now cover each staircase in turn with some of your puzzle pieces. Record your solutions on graph paper. For the last staircase, you will need all of your pieces.

18. Make a list of all the rectangles, including squares, having area 28 or less. Their dimensions (length and width) should be *whole numbers greater than 1*. (In other words, the shortest sides should be 2.) There are 25 such rectangles.

19. Draw these rectangles, and use the puzzle pieces to cover them. Record your solutions on graph paper. (It is impossible to cover one of the rectangles.)

POLYOMINO AREA AND PERIMETER

Think of the monomino. Its area is 1 and its perimeter is 4. Think of the domino. Its area is 2 and its perimeter is 6.

20. **Exploration** Is the number representing the perimeter of a given polyomino always greater than the number representing its area, or can it be equal to it, or even smaller? Look over your notes and sketches from Lesson 2, and experiment some more on graph paper if you need to. Then write a paragraph to answer this question fully, with examples and graph paper illustrations.

21. Find out if there are polyominoes having both area and perimeter equal to

a. 14 b. 15 c. 16

d. 17 e. 18 f. 20

WORD SQUARES

This is a word square.

	a	b	c	d
a	M	A	T	H
b	A	C	R	E
c	T	R	E	E
d	H	E	E	L

Note that the words can be read across or down. The largest word square in the English language took years of hard work to discover. It is made up of obscure ten-letter words.

22. How many letter tiles are used in the word square above?

23. Make a table showing the number of tiles required for word squares, extended to word length 10.

Word Length	Tiles
1	1
2	4
3	9
4	...

24. The numbers you found in problem 11 (1, 4, 9, ...) are called the *square numbers.* Explain how they can be calculated.

25. There is an interesting pattern based on adding pairs of consecutive triangular numbers (1 + 3, 3 + 6, ...) Explain it.

26. Draw a sketch of the third triangular number put together with the fourth triangular number (upside down) to show a square number.

27. What do you think the 100^{th} triangular number and the 101^{st} triangular number add up to?

28. Make a word square using these clues. The answer words are all four letters long and can be read both across and down.

a. Made to be played.

b. You learned about it in this chapter.

c. Don't make one!

d. A piece of cake.