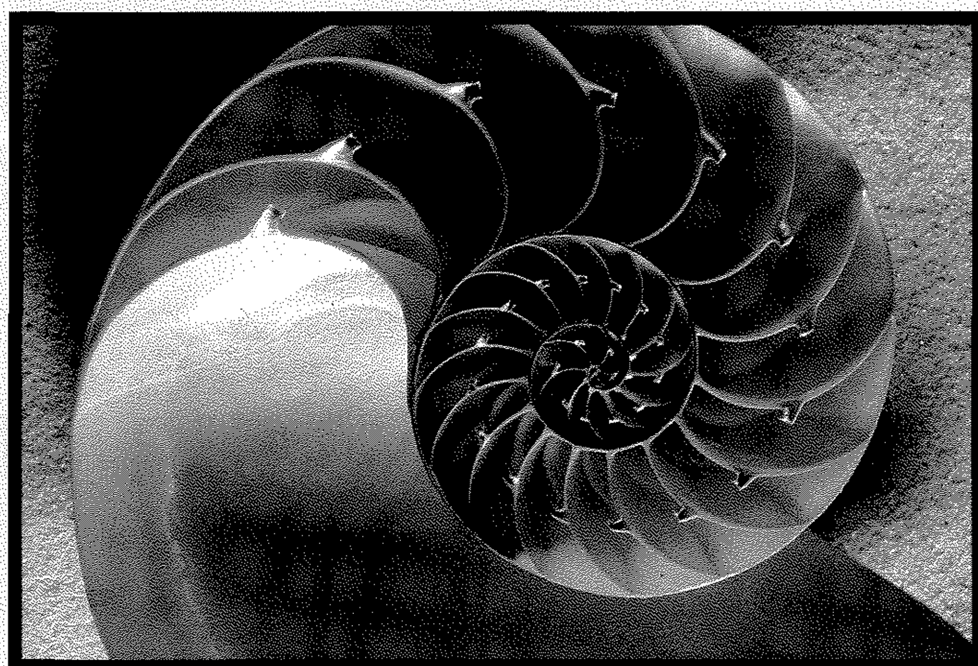
CHAPTER 2



The equiangular spiral of a nautilus shell

Coming in this chapter:

Exploration 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...
There are many patterns in this sequence of numbers. Find as many as you can, using addition, subtraction, multiplication, and division.

OPERATIONS AND FUNCTIONS

LESSON 2.1

Minus and Opposites

You will need:

the Lab Gear 

THREE MEANINGS OF MINUS

The *minus* sign can mean three different things, depending on the context.

- It can mean **negative**. In front of a positive number, and only there, it means negative. Example: $^-2$ can mean negative 2.
- It can mean **opposite**. The opposite of a number is what you add to it to get zero. Example: $^-2$ can mean the opposite of 2, which is negative 2, since $2 + {}^-2 = 0$. Likewise, ^-x means the opposite of x, and $x + {}^-x = 0$.
- It can mean **subtract.** Between two expressions, it means subtract the second expression from the first one. For example, $x - 3$ means subtract 3 from x.

1. For each of the following, write an explanation of what the minus sign means.

a. $y - 5$ b. $^-(5x + 1)$

c. $^-2$ d. ^-x

2. Write the value of ^-x if:

a. $x = 2$; b. $x = {}^-3$.

3. True or False? (Explain your answers.)

a. ^-x is always negative.

b. ^-x can be positive.

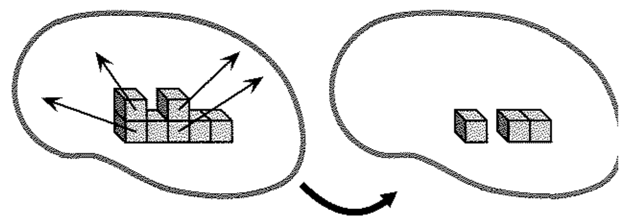
Notation: In this book, the minus sign meaning *negative* or *opposite* will be smaller than the one for subtract. In handwriting, this is not necessary. However some calculators use different keys for the two meanings: [−] for subtraction, and [(−)] or [+/−] for *negative* or *opposite*.

There are two ways of showing minus with the Lab Gear: upstairs and the minus area.

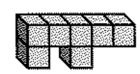
UPSTAIRS

Rule: Any blocks placed on top of other blocks are preceded by a minus sign.

This figure shows $5 - 2$. Notice that the *uncovered* part of the bottom block equals 3. If you remove matching upstairs and downstairs blocks, you will be left with three downstairs blocks. This is how we show $5 - 2 = 3$ with upstairs and downstairs blocks.

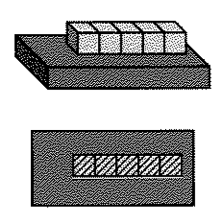


This figure shows $2 - 5$. If you mentally remove matching blocks downstairs and upstairs, you are left with 3 upstairs blocks, or $^-3$. We can only do this mentally, however, since blocks cannot float in mid-air.

 $2 - 5 = {}^-3$

Do not stack Lab Gear blocks more than two levels high. Two levels are enough to illustrate many ideas of algebra and will keep things clear. More would be confusing.

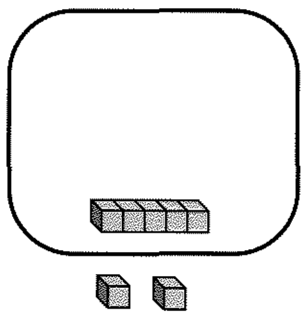
Subtraction with variables is shown in the same way. The amount being subtracted must be placed upstairs. Note that upstairs blocks are shaded in the 2-D sketch.



The upstairs method of showing minus is important and useful, but it is limited; it cannot easily be used to show minus when it means *negative* or *opposite.*

THE MINUS AREA

Look at your workmat. The rectangles with rounded corners represent the **minus areas**. The whole collection of blocks inside the minus area is preceded by a minus sign. For example, $2 - 5$ can be shown this way. (Here the minus sign means *subtract.*)



If you remove the matching blocks inside and outside the minus area, you will be left with three blocks inside the minus area, or $^-3$. (Here the minus sign means *negative.*)

4. Sketch how you would show each quantity on the workmat. You may need to use upstairs in some of the problems.

a. $5 - x$ b. $x - 5$

c. $^-(x + 5)$ d. $^-(5 - x)$

e. $^-5$

5. Summary

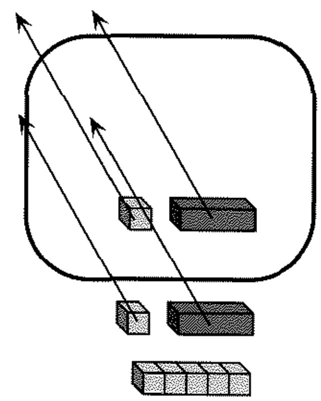
a. Explain, using examples, how the minus area can show all three meanings of minus.

b. Which of the three meanings does the upstairs method show best? Explain.

c. Put some blocks in the minus area, including some blocks upstairs. Sketch. What quantity does this arrangement represent?

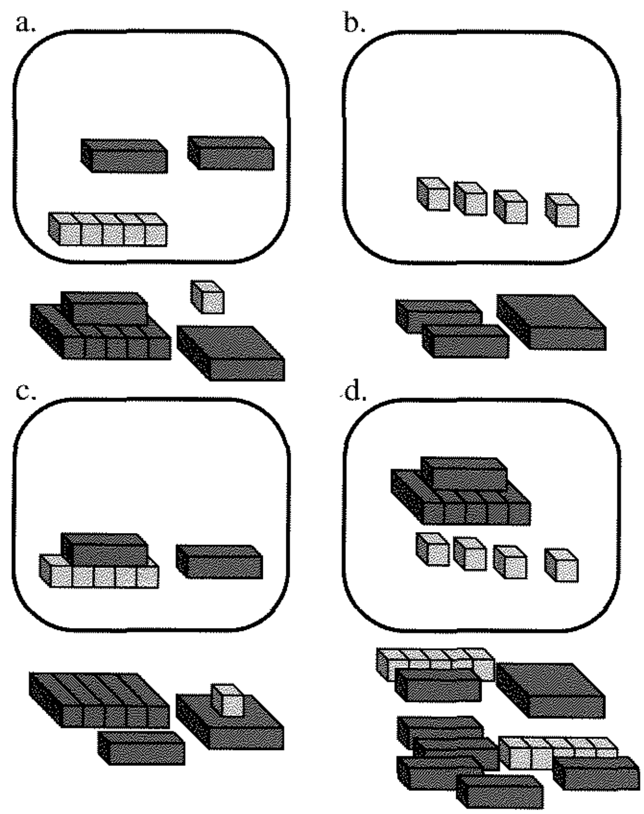
REMOVING OPPOSITES

When the quantities inside and outside the minus area are the same, they add up to zero and can be removed. For example, the figure shows that $5 + x + 1 - (x + 1) = 5$.



Similarly, matching upstairs and downstairs quantities add up to zero, and can be removed.

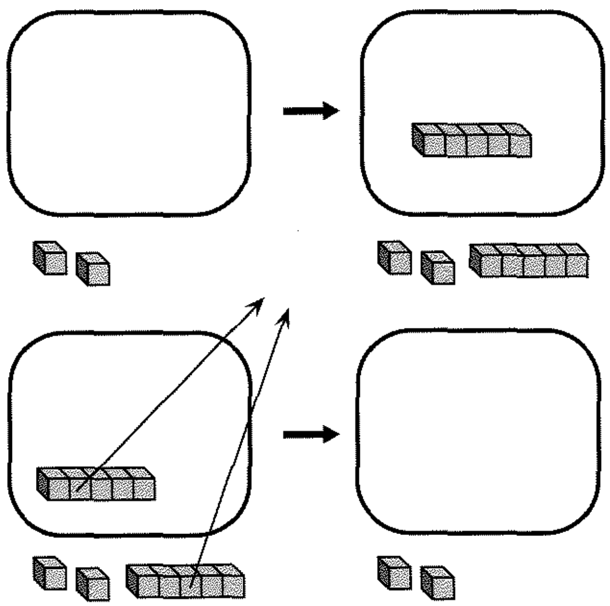
6. Two of these four figures represent the same quantity. Which two? Explain.



ADDING ZERO

The number 2 can be shown most simply with two 1-blocks outside the minus area. However, sometimes it is useful to show the number 2 using more blocks.

For example, after adding a five-block in the minus area and a five-block outside, the figure still shows 2. Since 5 and -5 are opposites, their sum is zero, so we really added zero. The technique of adding zero is useful in many situations.



7. Sketch two other ways to show the number 2.

8. Sketch or explain how to show -9 with:
   a. three blocks; b. five blocks;
   c. seven blocks.

9. Sketch or explain how you would show 5 with:
   a. 3 blocks; b. 11 blocks.

10. Can you show 5 with any number of blocks? Can you show it with 100 blocks? With 101 blocks? Explain your answers.

11. a. Show $x - 1$ in at least three different ways. Sketch or explain.
    b. Show $1 - x$ in at least three different ways. Sketch or explain.

MINUS PUZZLES

12. Nineteen numbers can be shown with exactly two yellow blocks. What are they?

13. Find three ways to show -4 using only a 5-block and a 1-block. Sketch or explain.

14. Find four ways to show 3 with three blocks. Sketch or explain.

15. Find four ways to show -8 with four blocks. Sketch or explain.

16. Make up a puzzle like the above for a classmate. Solve a classmate's puzzle.

LESSON 2.2

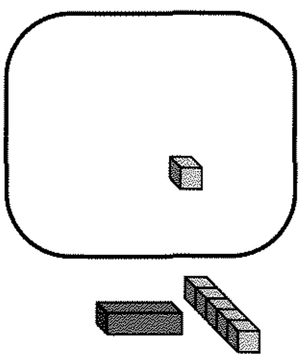
Adding and Subtracting

You will need:

the Lab Gear 

ASSOCIATIVE AND COMMUTATIVE LAWS

As you know, addition can be modeled with the Lab Gear by putting together collections of blocks on the workmat. For example, $x + 5$ means *put together x and 5* and $(x + 5) + -1$ means *put together* $x + 5$ *and* -1. This expression can be simplified by removing opposites, which would give us $x + 4$.

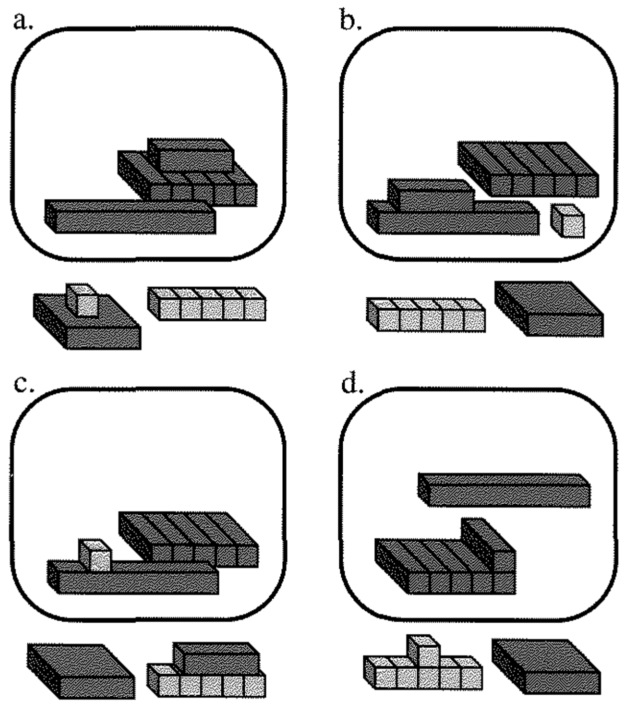


Note that the same figure could have been used to represent $x + (5 + -1)$. This is because, in an addition, quantities can be grouped in any way. This is called the *associative law for addition.*

The same figure could have been used to represent $-1 + (x + 5)$, or $(5 + x) + -1$. This is because in an addition, you can change the order of the terms. This is called the *commutative law for addition.*

Finally, because of the commutative and associative properties, the -1 could have been shown upstairs on top of the x, or on top of the 5, instead of in the minus area. In every case, the expression would simplify to $x + 4$.

1. After simplifying these expressions, one will be different from the rest. Which one? Explain.



Add these polynomials. (In other words, remove opposites and combine like terms.) It may help to use the Lab Gear.

2. $(xy + 3x + 1) + (2x + 3)$

3. $(xy - 3x + 1) + (-2x - 3)$

4. $(xy + 3x - 1) + (-2x + 3)$

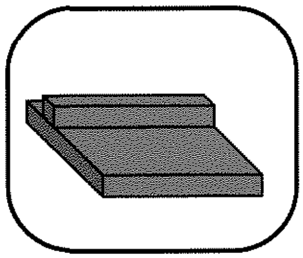
5. $(3 - 2x + xy) + (3x - 1)$

6. What do you notice about problems 4 and 5? Explain.

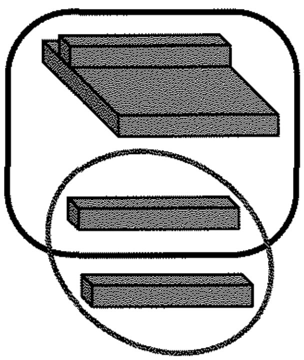
UPSTAIRS BLOCKS IN THE MINUS AREA

Here is a useful technique. To simplify upstairs blocks in the minus area, you can add zero, then remove opposites. For example, this figure shows how to simplify

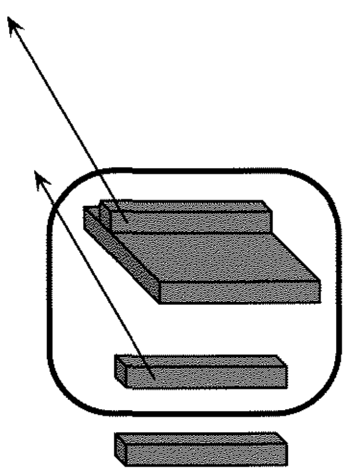
$$-(y^2 - y).$$



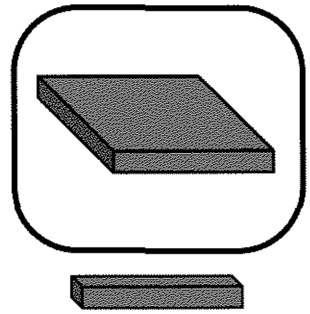
- **Add zero** by adding y inside and outside the minus area.



- **Remove opposites,** the matching blocks upstairs and downstairs.



- The simplified form is $-y^2 + y$. All the blocks are downstairs.



When working with the Lab Gear on the workmat, *simplifying* usually means

- removing opposites;
- combining like terms; and
- getting everything downstairs.

7. Model each expression using the Lab Gear. You will have to use both the minus area and upstairs blocks. Then simplify.

a. $-(5 - x)$ b. $-(x - 5)$
c. $3 - (x - 2)$ d. $(x - 2) - 3$

For problems 8–11 below:

- Build the first expression with the Lab Gear on the left side of the workmat.
- Next, compare each of the expressions a, b, c, and d to the original expression. (To make the comparison, build the expression on the right side of the workmat and simplify as needed.)

8. Which of these expressions are equivalent to $-(x + y)$?

a. $-x + (-y)$ b. $-x - y$
c. $-x + y$ d. $y - x$

9. Which of these expressions are equivalent to $-(x - y)$?

a. $-x + y$ b. $-x - y$
c. $-(y - x)$ d. $y - x$

10. Which of these expressions are equivalent to $-(y - x)$?

a. $x - y$ b. $-x + y$
c. $-y + x$ d. $-y - x$

11. Which of these expressions are equivalent to $-(-x + y)$?

a. $-x + y$ b. $-y - x$
c. $x - y$ d. $y - x$

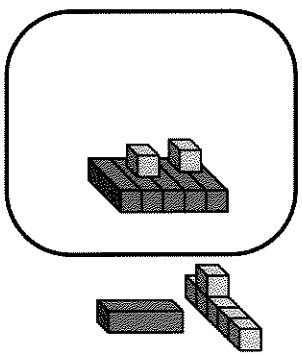
12. Generalization For each expression below, write an equivalent one without parentheses. Do not use the Lab Gear.

a. $-(a + b)$ b. $-(a - b)$
c. $-(-a + b)$

SUBTRACTION

The figure shows the subtraction

$$(x + 5 - 1) - (5x - 2).$$



13. Use what you learned in the previous section to simplify it.

14. Simplify, using the Lab Gear.

a. $x - (5x + 2)$ b. $x - (5x - 2)$

15. Simplify, with or without the Lab Gear.

a. $(6x + 2) - (3x + 1)$
b. $(3x - 2) - (6x + 1)$
c. $(6x - 1) - (3x - 2)$
d. $(3x - 2) - (6x - 1)$

16. In (a-c) find the missing expression. It may help to use the Lab Gear.

a. $-3x -$ _____ $= -4x$
b. $-3y -$ _____ $= -6y$
c. $-3y -$ _____ $= -2x - 4y$

17. Summary

a. Write a subtraction problem that you could model with the Lab Gear by putting blocks upstairs in the minus area.
b. Simplify this subtraction without using the Lab Gear. Explain the rule you are using.

18. How could you show the subtraction

$$y - -x$$

with the Lab Gear? (Hint: Remember about adding zero.) What would it look like after it is simplified? What is a rule you could use without the blocks to simplify this kind of expression?

19. Simplify without the blocks, $-(-a - b)$. Explain your answer.

REVIEW MINUS PUZZLE

20. a. Using the Lab Gear, show -4 in five different ways.
b. What numbers of blocks can and cannot be used to show -4?

LESSON 2.3

Multiplying

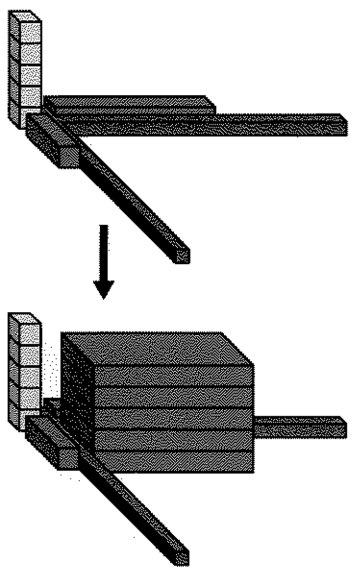
You will need:

the Lab Gear 

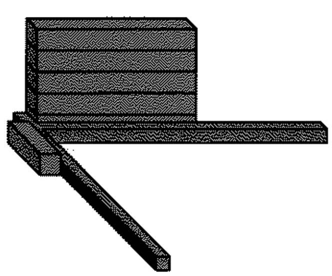
THREE DIMENSIONS

Just as we used the area of a rectangle to help us model multiplication of two factors, we can use the volume of a box to help us model multiplication of three factors.

For example, $5 \cdot x \cdot y$ can be shown like this.



But another way to show it could be:



1. Use the Lab Gear to show how x^2y can be seen as a product of:

a. three factors;

b. two factors;

c. two factors in another way.

ASSOCIATIVE AND COMMUTATIVE LAWS

In a multiplication the factors can be grouped in any way. For example, $(-2 \cdot 3) \cdot 4 = -2 \cdot (3 \cdot 4)$. This is called *the associative law for multiplication.*

In a multiplication the factors can be multiplied in any order. For example, $5 \cdot (-6) = (-6) \cdot 5$. This is called *the commutative law fo multiplication.*

2. Using six xy-blocks, it is possible to mak a rectangle in four different ways. Find al four rectangles, and write a multiplicatio equation for each.

3. Using six xy-blocks, it is also possible to make a three-dimensional box. There are many such boxes. Find five, and write at least two multiplications for each one.

4. **Summary** Explain how problems 2-3 about $6xy$ provide examples of the associative and commutative laws for multiplication.

HOW MANY TERMS?

5. **Exploration** After combining like terms, how many terms does the product have for each of the following multiplications? Is there a pattern? You may use the Lab Gear.

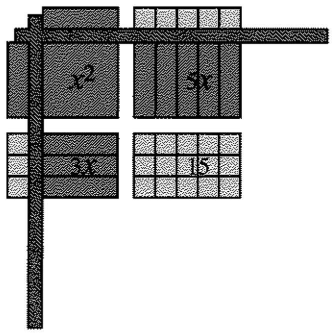
a. $2x \cdot 3x$

b. $2(x + 3)$

c. $2x(x + 3x)$

d. $(3 + x)(x + 2)$

The figure shows $(x + 3)(x + 5)$.



The resulting rectangle is made up of four smaller rectangles. The area of each one is shown in the figure.

6. a. Which two rectangles are made up of the same kind of block?

b. What is the answer to the multiplication $(x + 3)(x + 5)$? Combine like terms in your answer. How many terms are in your final answer?

7. a. Use the corner piece to model the multiplication $3x(x + 5)$. Sketch it, showing the resulting rectangle.

b. On your sketch, write the area of each of the smaller rectangles that make up the larger rectangle.

c. Write the result of the multiplication $3x(x + 5)$. Combine like terms.

d. How many terms are in your final answer?

8. Repeat problem 7 for

$$(x + 3)(x + y + 5).$$

9. Repeat problem 7 for

$$(x + y + 3)(x + y + 5).$$

10. Use the Lab Gear to model a multiplication problem that has four terms in the final answer. Sketch the blocks and write the multiplication.

MAKE A RECTANGLE

Take blocks for each expression.

a. Arrange them into a rectangle.

b. Write a multiplication equation of the form *length times width equals area.*

11. $xy + 5y$ **12.** $xy + 7x$

13. $7y + 7x$ **14.** $x^2 + 7x$

15. $x^2 + 7x + xy$

16. Do not use the Lab Gear for this problem. Write the addition

$$y^2 + 2xy + 3y$$

as a multiplication. Explain how you solved the problem.

In problems 17 and 18, take blocks for each expression.

a. Arrange them into a rectangle.

b. Write a multiplication equation of the form *length times width equals area.*

17. $x^2 + 7x + 6$

18. $x^2 + 7x + 10$

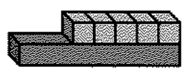
LESSON

2.4 The Distributive Law

LINEAR ADDITION AND SUBTRACTION

In the case of x, y, and constant blocks — in other words quantities of degree 1 or 0 — you can think of adding as putting together blocks end-to-end *in a line*. For example, $2x + 5$ is shown by connecting the two x-blocks and the 5-block on their 1-by-1 faces.

Similarly, subtraction of quantities of degree 0 and 1 can be shown linearly, by making sure that the uncovered area models a single line segment. The figure shows $y - 5$.



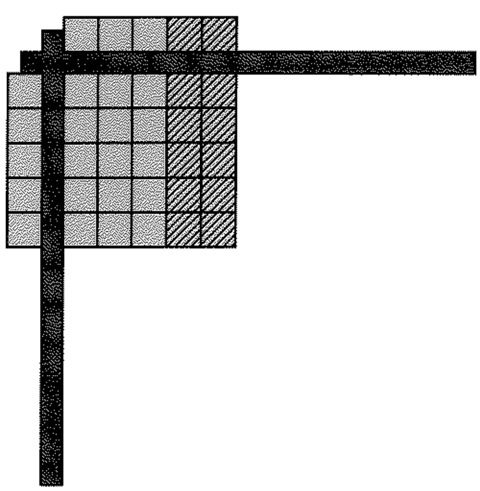
This representation is based on a *length* model of addition and subtraction.

1. Sketch these sums, showing length.
a. $y + 2$ b. $3x + 1$

2. Sketch these differences, showing length.
a. $y - 2$ b. $3x - 1$

THE UNCOVERED RECTANGLE

It is possible to use the corner piece for multiplication when minus signs are involved. For example, this figure shows the multiplication $5(5 - 2)$.



Remember that the shaded blocks are upstairs Look at the part of the downstairs blocks that are not covered by upstairs blocks. The answe to the multiplication is represented by the **uncovered rectangle** with dimensions 5 and $5 - 2$. Of course, the product is 5 times 3, or 15, which is the answer you get when you sim plify upstairs and downstairs blocks.

THE DISTRIBUTIVE LAW

Find these products, using the Lab Gear. Remember to use upstairs for minus.

3. $x(5 + y)$ **4.** $(5 - x)y$

5. $5(x + y)$ **6.** $(y - 5)x$

7. $y(5 + x)$ **8.** $(y - x)5$

9. **Summary** Explain how you can correctly remove parentheses from an algebraic expression when they are preceded or followed by a multiplication, and when there is more than one term in the parentheses.

10. Remove the parentheses.
a. $a(b + c)$ b. $(a - b)c$

The rule you have discovered in this section is called *the distributive law of multiplication over addition and subtraction.*

Use the distributive law to multiply. You may use the Lab Gear to check your work.

11. a. $2x(x + 1)$ b. $2x(x - 1)$

12. a. $2x(x + y + 5)$
b. $2x(x + y - 5)$
c. $2x(-x + y + 5)$
d. $2x(x - y + 5)$

For problems 13-18:

a. Show the quantity with the Lab Gear, using upstairs to show minus.
b. Arrange the blocks so the uncovered part is a rectangle.
c. Write a multiplication of the type, *length times width* = *area* for the uncovered rectangle.

13. $xy - 2y$ **14.** $xy - 2x$

15. $xy - x^2$ **16.** $xy + x - x^2$

17. $y^2 + xy - 5y$ **18.** $y^2 - xy - y$

19. Explain how someone might have done problem 18 without the Lab Gear.

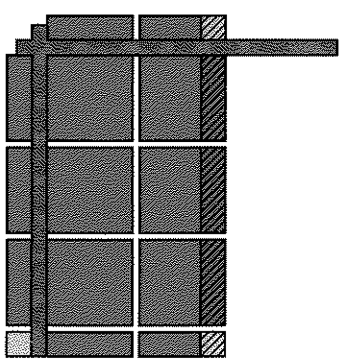
20. Write $x^2 - xy - x$ as a multiplication of the type, *length times width* = *area,* for the uncovered rectangle.

RELATED PRODUCTS

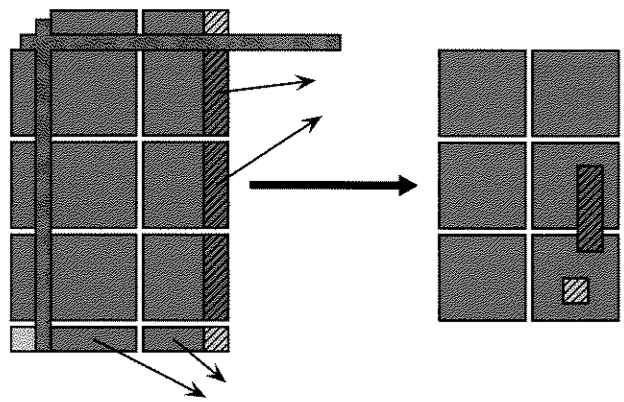
21. Use the corner piece to show

$$(3x + 1)(2x - 1).$$

This figure shows the product $(3x + 1)(2x - 1)$.



Notice that, inside the corner piece, the uncovered rectangle has dimensions $3x + 1$ and $2x - 1$. These are the original factors. This tells you that we did the multiplication correctly. But the product can be simplified, as shown below.



22. a. Explain what was done to the blocks in problem 21 after using the corner piece. Which blocks were removed, and why?

b. Write the final answer, combining like terms.

23. Use the Lab Gear to find the product: $(3x - 1)(2x + 1)$. Sketch the process as was done for problem 21.

24. a. Show the multiplication $(3x + 2)(2x + 5)$ with the Lab Gear. Write the product.

b. Write two more multiplications, both involving minus, that use the same blocks as $(3x + 2)(2x + 5)$. In each case write the product.

25. Summary You can use the same blocks to show all three of these products with the Lab Gear. Explain why the products are different, even though the same blocks are used. Include sketches as part of your explanation.

a. $(2x + 3)(3x + 5)$

b. $(2x + 3)(3x - 5)$

c. $(2x - 3)(3x + 5)$

26. You will learn how to model

$$(2x - 3)(3x - 5)$$

with the Lab Gear in a later chapter. Try to find a way to do this without looking ahead in the book.

REVIEW ***UNLIKE TERMS***

27. Al *still* doesn't like terms. For each problem, give the correct answer, if possible, and explain what Al did wrong. Use Lab Gear sketches or substitute numbers.

a. $x^2 - x = x$

b. $3x - x = 3$

c. $9x - 4y = 5(x - y)$

THINKING WRITING

2.A Operations

The teacher had just returned the math test, and no one was looking very happy. Martin had missed *all* the problems.

Test Name: *Martin P.*

Operations

1. $2^3 = 6$
2. $3x + x = 3x^2$
3. $2x^3 - x^2 = x$
4. $5 - 2x = 3x$
5. $4 - 2 \cdot 6x = 12x$
6. $(2x - 3) - (x - 2) = x + 5$
7. $6x - (x^2 - 4x) = 2x - x^2$
8. $-(y^2 - x^2) = -y^2 - x^2$
9. $(2x + 1)(3x - 5) = 6x^2 - 5$
10. $2x(-y + 5) = 2x - y + 5$
11. $2y + 3x = 5xy$
12. $6 - 2(x + 3) = 4x + 12$

"I hate math tests," Martin groaned. "I'd rather have my teeth pulled out." Mary would not show her test to anyone, but she looked miserable, too. "I'll need a brain transplant to pass this course," she moaned. Lew, the math whiz, grimaced at his test score and glared at his crutches. He was used to getting everything right, but he had just had an operation on his knee after an injury on the playing field. Math had been the last thing on his mind when he took the test.

Then the teacher did an unusual thing. He handed out these instructions:

Free Points!

You can get extra points on the **Operations** test if you can correct your mistakes. This is what you need to do:

a. For each problem, explain your mistake. Try to figure out what you were thinking. Most of your mistakes have to do with operations.

b. Show me you now know how to do the problem correctly. Use sketches of the Lab Gear or explain a rule you have learned. Don't just give me the answer.

c. Finally, write the correct answer to the problem.

What should Martin write to get his free points? Write out the corrections for him.

LESSON

2.5 Powers

DOING DISHES

Abe agreed to do the dishes daily in exchange for one cent on April 1st, two cents on April 2nd, four cents on April 3rd, and so on, doubling the amount every day.

1. To find out how much money Abe was earning, make a table like this one, for at least the first ten days.

Day #	Cents	Total
1	1	1
2	2	3
3	4	7
4	...	...

2. How are the numbers in the *Cents* column calculated?

3. How much money did Abe get paid on April 30? Explain how you figured out the answer. Do you think you could talk your parents into an arrangement like this?

4. a. Study the table, looking for a pattern in the *Total* column. Describe the pattern.

b. How much money did Abe make altogether during the month of April?

Definitions: Exponents

Exponentiation, or *raising to a power,* is the operation of multiplying a number by itself repeatedly. The number that is multiplied is called the *base.* The number of factors is called the *exponent.*

Examples:

- The expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is written 2^5, where 2 is the base and 5 is the exponent.
- You are already familiar with squaring an cubing, which are special cases of exponentiation in the case of raising to the sec ond and third powers.
- The numbers in the Cents column in the above table are called the *powers of 2,* because they can be obtained by raising 2 to different powers.

Notation:

- On calculators, it is not possible to use thi notation. Instead, 2^5 is entered as 2 [y^x] 5, 2 [x^y] 5, or 2 [^] 5.
- On computers, most word processors allow the user to type exponents (called *superscripts*).
- Computer programming languages use 2^5, 2**5, or `POWER 2 5`.

5. **Generalization**

a. How much money did Abe make on th n^{th} day of April? (Watch out.)

b. What is the number in the *Total* colum on day n? Explain.

EXPONENTIAL NOTATION

The number 64 can be written in exponential notation as 2^6 or 8^2. (Check this with your cal culator or by mental multiplication.)

6. Find another way to write 64 in exponential notation.

7. Write each of these numbers in exponential notation. Do not use 1 as an exponent If possible, find more than one way. It ma help to use your calculator.

a. 81 b. 1

c. 1024 d. 625

e. 6561 f. -512

CHAIN LETTER

Lara received this letter.

> *Dear Lara,*
>
> *Send copies of this letter to five people, or the most terrible bad luck will afflict you. One man broke the chain, and a flower-pot fell on his head, giving him a terrible headache which continues to this day.*
>
> *Don't look a gift-horse in the eye. Rome was not built in a pond. Don't cry over spilt tears.*
>
> *Please do not break the chain! It was started in 1919 by a psychic.*
>
> *Bea*

Assume that the chain is not broken, and that each person who receives it takes a week to send out five copies.

8. After one week, five people receive Lara's letter. After another week, how many people receive the letter? Make a table like the following for the first ten weeks.

Week #	Letters received this week	Total number received so far
1	5	5
2	25	30
3	...	...

9. How many weeks until the number of letters received that week is greater than the population of the United States?

10. How many letters were received in the n^{th} week?

11. If each person made six copies of the letter instead of five, how would your answer to problem 10 change?

12. Do you think that the chain was started in 1919? Explain why or why not.

13. How do the assumptions we made to solve this problem compare with what happens in the real world with chain letters?

GETTING HELP

Assume Lara gave a copy of the letter to Lea and they each sent five copies in the first week.

14. If everything continues as in the previous section, how many people receive the letter? Make a table like the following for the first five weeks.

Week #	Letters received this week	Total number received so far
1	10	10
2	50	60
3	...	...

15. Write the number of letters received in the 10^{th} week as an expression *using exponents.*

16. How many letters were received in the n^{th} week?

17. If each person asked a friend to help in the same way, how would your answers to problems 14-16 change?

LESSON 2.6

Finding Patterns

You will need:

graph paper

PARKING RATES

Two downtown parking garages charge different amounts, as shown by the following signs.

Ball Garage	
up to:	'U' pay:
1/2 hour	35 cents
1 hour	70 cents
1 1/2 hr	$1.05
2 hours	$1.40
3 hours	$2.65
4 hours	$3.90
5 hours	$5.15
6 hours	$6.40
7 hours	$7.65
all day	$8.90

Bear Garage	
up to:	fee:
1 hour	$1.05
2 hours	$2.10
3 hours	$3.15
4 hours	$4.20
5 hours	$5.25
6 hours	$6.30
all day	$7.25

1. If you park for two hours and five minutes, you have to pay the three-hour fee. How much is that at each garage?
2. People who work downtown tend to use one of the garages, and people who shop there tend to use the other. Explain why, with examples.
3. Lara notices that for the amount of time she is planning to park, the cost differen between the two garages is less than a quarter. How long is she planning to par
4. The parking fees at the Bear Garage mostly fit a pattern. Describe the pattern in words. Where does it break down?
5. The parking fees at the Ball Garage fit a more complicated pattern. Describe the pattern in words. Why might the owner Ball Garage have chosen a complicated pattern?

Analyzing numbers can be useful in making intelligent decisions. Here is an example.

6. Zalman owns an empty lot. He decides t convert it to a parking garage. He wants charge a fee that is not too expensive. H decides on these rules:
   - The fee should increase by a constant amount for each half-hour.
   - For parking times from a half-hour to nine hours, the fee should never be m than 25 cents higher than either Ball's Bear's fee.
   - The fee should be the highest possible fee that satisfies these rules.

   a. Explain why Zalman might have cho sen each rule.

   b. What rate should he choose? (For co venience in making change, it should a multiple of 5 cents.) Explain.
7. Graph the parking fees for all three garages. Put *time* on the horizontal axis, and *cost* on the vertical axis.

FIBONACCI SEQUENCES

The following numbers are called *Fibonacci numbers* after the Italian mathematician who first studied them:

1, 1, 2, 3, 5, 8, 13, 21...

8. Describe the pattern. Then give the next five Fibonacci numbers. (As a hint, if you have not yet discovered the pattern, look at the *Lucas numbers* — named after another mathematician — which follow the same principle: 1, 3, 4, 7, 11, 18, 29, 47, 76, 123...)

9. **Exploration** Look for patterns in the Fibonacci numbers. You may use addition, subtraction, or multiplication.

Definition: A *sequence* is an ordered list of numbers or expressions.

10. You can create your own Fibonacci-like sequence. Choose any two numbers, and use them as the starting values for a sequence like the ones described in problem 8. Name the sequence after yourself. Have a classmate check that your sequence is correct.

11. a. Find the first ten terms in a new sequence by adding the Fibonacci and the Lucas numbers. (The sequence should start: 2, 4, 6, 10, 16...) Is the resulting sequence a Fibonacci-like sequence? (Does it follow the same rule?)

b. Find the first ten terms in a new sequence by subtracting the Fibonacci numbers from the Lucas numbers. Compare your answer to the one in (a).

c. Find the first ten terms in a new sequence by dividing the sequence in (b) by 2. The result should be familiar.

12. Look for odd/even patterns in Fibonacci-like sequences including the original one, the Lucas sequence, and three named after students in your class. Explain.

13. Extend the Fibonacci and Lucas sequences to the left. In other words, what number should come before the first number? What number should come before that, and so on? Describe the resulting patterns.

MISSING NUMBERS

The following Fibonacci-like sequence fragments have numbers missing. Copy the sequences and fill in the blanks.

14. a. 0.5, 1.1, ___, ___, ___
b. 5, -4, ___, ___, ___
c. -6, -7, ___, ___, ___

15. a. ___, ___, ___, 11, 20
b. 2, ___, 7, ___, ___
c. ___, 3, ___, 9, ___

You may need to use trial and error for these.

16. a. 1, ___, ___, 11, ___
b. 12, ___, ___, 13, ___
c. ___, 8, ___, ___, 10

17. a. 1, ___, ___, ___, 11
b. 1, ___, ___, ___, 20
c. 2, ___, ___, ___, 19

18. a. 3, ___, ___, ___, ___, 29
b. 5, ___, ___, ___, ___, ___, 17

USING VARIABLES

19. Look at problem 17. Describe the relationship between the middle number and the outer numbers.

20. Create a five-term Fibonacci-like sequence in which the first two terms are x and y.

21. Check whether the pattern you noticed in problem 19 works for the sequence you just created. Explain.

22. Fill in the blanks for this Fibonacci-like sequence. -123, ___, ___, ___, 456

23. Extend the sequence you started in problem 20. Look for patterns.

FIBONACCI PUZZLE

24. How many Fibonacci-like sequences can you find that involve only positive whole numbers and include your age *in fourth place or later?* How about your teacher's age, or the age of a parent or adult friend

DISCOVERY ***PERIMETER ARRANGEMENTS***

25. **Exploration** Make sketches of some different ways that you could put together an x-block and an x^2-block in two dimensions. (They have to touch each other, but they don't have to make a rectangle.) Use your imagination. There are more than two arrangements possible. Is it possible to sketch all the arrangements you think up?

26. Find the perimeters of the arrangements you sketched in problem 19. Write each perimeter next to the sketch. Make sure you have found the largest and smallest perimeters possible.

27. Find two arrangements that have the same perimeter, but look as different from each other as possible.

REVIEW ***MISSING TERMS***

28. What terms are missing? More than one term may be missing in each problem.

a. $3x^2 - 4x +$ ___ $= -9x^2 + 8x + 7$

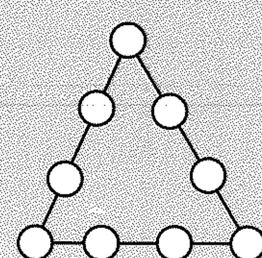
b. $-x^2y + 6xy +$ ___ $= 9x^2y + 8y$

c. $3x^2 - 4x - ($ ___ $) = -9x^2 + 8x + 7$

d. $-x^2y + 6xy - ($ ___ $) = 9x^2y + 8y$

PUZZLE ***MAGIC TRIANGLE***

29. Put an integer from -4 to 4 in each circle to get equal sums along each side of the triangle. Find as many different sums as you can.



2.7 Functions and Function Diagrams

You will need:
- graph paper
- function diagram paper

FUNCTIONS FROM IN-OUT TABLES

Definition: The following tables are called input-output tables, or *in-out tables.*

The number that is put in is x, and y is the number that comes out. Each table has a rule that allows you to get y from x. For example, the rule for the table in problem 1 is *to get y, add three to x.* We say that y can be written *as a function of x:* $y = x + 3$.

Definition: A *function* is a rule that assigns a single output to each input.

For each of the following problems:

a. Copy the table.
b. Describe the rule that allows you to get y from x.
c. Use the rule to find the missing numbers. (In some cases, the missing numbers may be difficult to find; use trial and error and a calculator to make it easier.)
d. Write y as a function of x.

1.

x	y
-5	-2
7	10
5	
	-7

2.

x	y
7	3.8
10	6.8
0	
	10

3.

x	y
5	20
3	12
1	
	-1

4.

x	y
7	40
1	16
-2	4
-5	
	-12

5.

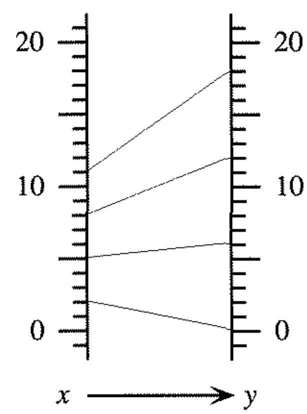
x	y
3	8
4	13
1	-2
7	
	20

6.

x	y
5	15
2	-6
-1	-9
6	
	54

7. **Exploration** Find as many functions as possible that assign the y value 4 to the x value 1.

FUNCTION DIAGRAMS



The figure above shows a function diagram for this table.

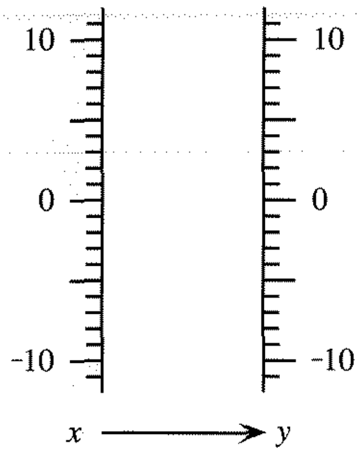
x	y
2	0
5	6
8	12
11	18

8. What is the function illustrated in the previous function diagram?

For each function in problems 9-12:

a. Make a table, using at least five in-out pairs.

b. Make a function diagram, using the scale shown below.



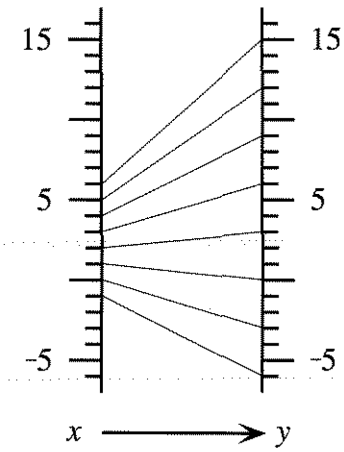
9. $y = x + 2$ 10. $y = x - 2$

11. $y = 2x$ 12. $y = x/2$

13. Make a function diagram for each of the tables in problems 1, 2, and 3. You will have to decide what scale to use on the x- and y-number lines. (For each problem, use the same scale on both number lines.)

Function diagrams are an important way of understanding functions. We will use them throughout this course.

I SEE WHERE YOU'RE COMING FROM



The following problems are about the above function diagram. Assume that more in-out lines could be added, following the same pattern.

14. Find the output when the input is:
a. 0   b. 5   c. -5

15. Find the output when the input is:
a. 99   b. -100   c. 1000

16. Find the output when the input is:
a. 1/2   b. 1/3   c. 1/6

For the following problem, you may need to use trial and error.

17. Find the input when the output is:
a. 0   b. 5   c. -5
d. 99   e. -100   f. 1000

UPS AND DOWNS

Each line in a function diagram connects an input point on the x-number line to its output point on the y-number line. We use the notation (x, y) to refer to such a line. Notice that in the previous diagram some of the lines go up, and some go down. For example: (5, 12) goes up, and (0, −3) goes down.

18. If you were to draw additional lines in the function diagram, could you correctly draw one that goes neither up nor down? Where would it start?

19. In describing the diagram, one might say 5 goes to 12, "moving" up 7 units. Which point "moves" down 5 units?

20. Find a point that moves

a. up 3 units; b. down 3 units;

c. up 6 units; d. down 4 units.

21. Use trial and error to find a point that moves

a. up 99 units;

b. down 100 units.

22. **Generalization** If you know of a point that moves up n units in the previous diagram, how would you find a point that moves down n units? Write a full explanation.

DISCOVERY *SURFACE AREA OF A BOX*

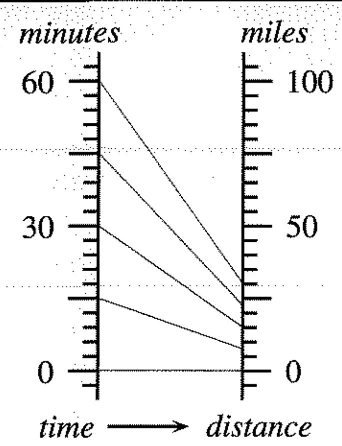
The volume of a box is given by the formula

$$volume = length \cdot width \cdot height.$$

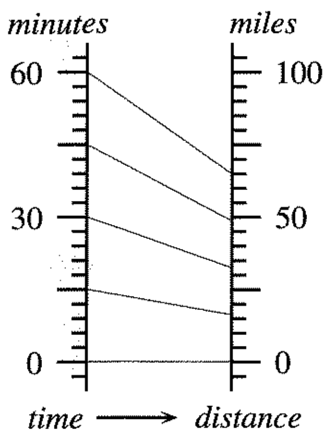
23. Write the surface area of a box as a function of length, width, and height. Compare your function with the ones found by some of your classmates.

2.8 Time, Distance, Speed

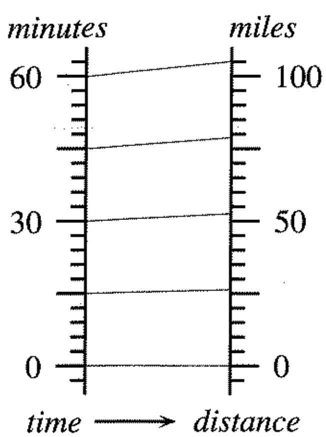
MOTION PICTURES



Roller Skater



Cheetah



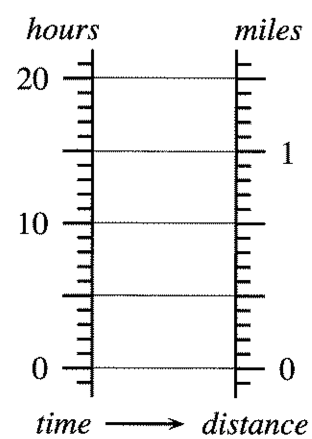
Needletail

The above function diagrams represent the motion of three living creatures: a fast roller skater; a cheetah (one of the world's fastest mammals, it's a large, wild cat that lives in Africa); and a white-throated needletail (one the world's fastest birds, it lives in Australia).

The diagrams assume that the three creatures ran a one-hour race, and were able to maintai their top speed for the full hour. (This is not realistic, but then neither is the idea of a rolle skater racing with a cheetah and a bird.)

Each diagram shows minutes on the x-numbe line, and miles on the y-number line.

1. Use the diagrams to estimate how far eac went in an hour.
2. After thirty minutes, approximately
   a. how far is the needletail ahead of the cheetah?
   b. how far is the cheetah ahead of the skater?
3. Estimate each speed
   a. in miles per hour;
   b. in miles per minute.
4. Explain how time-distance function diagrams allow you to compare speeds. Time is on the x-number line, distance is on the y-number line. Where is speed?



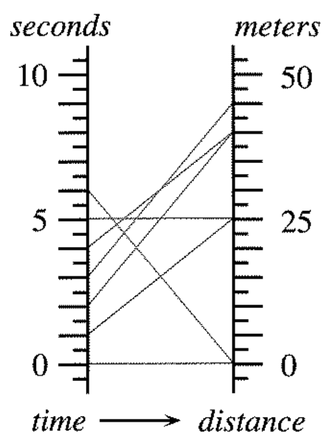
Sloth

5. The preceding diagram shows the hypothetical progress of a sloth. The x-number line represents time in hours, and the y-number line represents distance in miles. Compare the sloth's motion to the motion of the skater, cheetah, and needletail. How fast is it going per hour? Per minute?

6. Explain why someone comparing the sloth's speed to the needletail's might make a mistake and take the diagrams to mean the sloth is almost as fast as the needletail.

THE BALL

In a physics experiment, a ball is launched straight up by some device, and its height above the ground is recorded at one-second intervals. The resulting information is displayed in the function diagram below, where the x-number line represents time in seconds, and the y-number line represents distance from the ground at that time in meters.



Ball

On the function diagram, follow the motion of the ball with your finger on the y-number line, second by second.

7. During which one-second interval(s) did the ball move the fastest? The slowest?

8. At what time did the ball change direction?

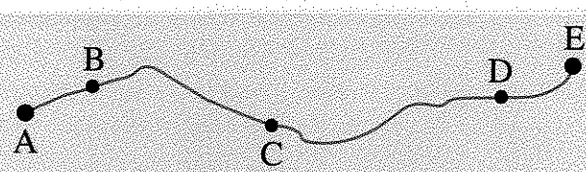
9. Make a table like this one, showing the height of the ball at one-second intervals. Extend the table until you have included all the information given on the function diagram.

Time (seconds)	Height (meters)
0	0
1	25
2	...

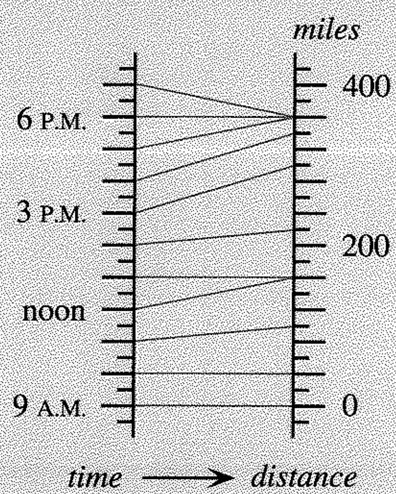
10. Estimate the times when the ball was at the following heights. (Give two times for each part, one on the way up, and one on the way down.)
    a. 40 m
    b. 30 m
    c. 20 m
    d. 10 m

THINKING
WRITING

2.B The Car Trip



A family is traveling by car from City A, in Cool County, towards City E. On this diagram, the x-number line represents the time of day, with 9 A.M. near the bottom, and 7 P.M. near the top; the y-number line represents distance from City A in miles.



Car Trip

1. **Report** Describe the trip as best you can from the information on the function diagram. In your paragraph, make clear what you get from the diagram and where you are making guesses to interpret the information. Your paragraph should include answers to the following questions, but should not be limited to them.
   - What time did the trip start?
   - What happened from 12 to 1? Where did it happen?
   - When did the family drive faster than the speed limit? How fast were they going then?
   - How could you explain the changes in speed that are evident from the diagran
   - What time did they arrive at their destination?
   - How far is City E from City A?

2. **Project**
   a. Using real towns and distances (perha taken from a road map), draw a map and a function diagram for another car trip.
   b. Get the map and function diagram tha one of your classmates made in part (a Write a paragraph describing the trip shown. Discuss your description with the person who made the map and dia- gram. Do you agree on what the figure convey? If you disagree, is one of you misinterpreting the figures? Or are boi interpretations correct?

LESSON 2.9

Operations and Function Diagrams

You will need:

- graph paper
- function diagram paper

ADDITION

1. Draw a function diagram to represent each of these functions.

a. $y = x + 6$ b. $y = x + 3$

c. Compare the two diagrams. How are they alike? How are they different?

The two function diagrams you just drew both represented functions of the form $y = x + b$, where b is a constant. In the first case, b was 6. In the second case, b was 3.

2. Draw three other function diagrams of the form $y = x + b$. Be sure to try at least one negative value of b.

3. a. Draw a function diagram for the function $y = x$.

b. The function $y = x$ is also of the form $y = x + b$. What is b?

4. The function diagrams you drew in problems 1-3 represent addition. In each case, to get the value of y, you added the number b to x. How are all of these diagrams alike? How are they different? How does the value of b affect the diagram?

MULTIPLICATION

5. Draw a function diagram to represent each of these functions.

a. $y = 2x$ b. $y = 3x$

c. Compare the two diagrams. How are they alike? How are they different?

The two function diagrams you just drew both represented functions of the form $y = mx$, where m is a constant. In the first case, m was 2. In the second case, m was 3.

6. Draw three other function diagrams of the form $y = mx$. Be sure to try at least one negative value of m and one value of m between 0 and 1.

7. The function $y = x$, for which you already have a diagram, is also of the form $y = mx$. What is m?

8. The function diagrams you just drew represent multiplication. In each case, to get the value of y you multiplied x by a number. How are all of these diagrams the same? How are they different?

9. Look at your multiplication diagrams. For each one, as the value of x increases from the bottom of its number line, follow the value of y on its number line with your finger.

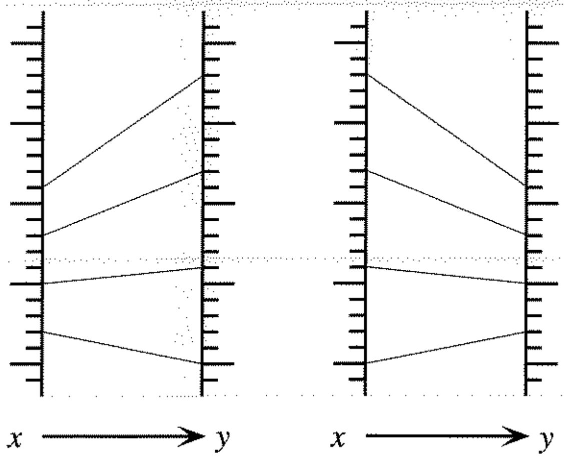
a. For what values of m does the value of y go up? Down?

b. Is there a value of m for which y goes neither up nor down, but remains unchanged?

c. For what values of m does the value of y change faster than x? More slowly?

d. Is there a value of m for which y changes at the same rate as x?

MIRROR IMAGE DIAGRAMS



The two function diagrams above are mirror images of each other.

10. Explain how to draw the mirror image of a function diagram.

For each of the following functions:

a. Draw the function diagram, using the same scale on the x- and y-number lines.
b. Draw the mirror image diagram.
c. Find the function corresponding to the mirror image.

11. $y = x + 3$ **12.** $y = 4x$

13. $y = x - 4$ **14.** $y = x/3$

15. Explain the relationship between the function corresponding to the mirror image and the original function.

16. **Report** Write a report summarizing what you learned in this lesson. Illustrate your report with examples of function diagrams. Your report should include, but n(be limited to, answers to the following questions:

- Addition can be represented by functions of the form $y = x + b$. What do their function diagrams look like if $b = 0$? What if b is greater than 0? Less than 0?
- Subtraction can be represented by func tions of the form $y = x - b$. How do their function diagrams compare with those of addition?
- Multiplication can be represented by functions of the form $y = mx$. What do their function diagrams look like if m i: negative? If m is positive? What if m is number between 0 and 1?
- Division can be represented by functio of the form $y = x/m$. How do their func tion diagrams compare with those of multiplication? What if m is positive? Negative? What if m is a number between 0 and 1?

17. Compare function diagrams of the form $y = b - x$ with those of the form $y = x - b$.

LESSON 2.10

Perimeter and Surface Area Functions

You will need:

the Lab Gear 

PERIMETER

1. Look at this sequence of block figures. Think about how it would continue, following the pattern. Then:

a. Sketch the next figure in the sequence.

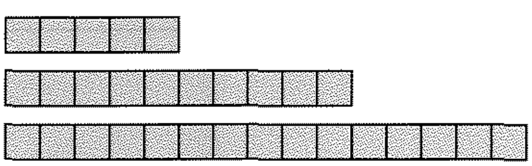
b. Copy and complete the table below.

c. Describe the pattern in words.

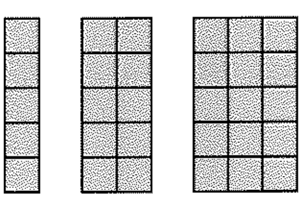
Figure #	Perimeter
1	4
2	6
3	8
4	...
10	...
100	...
n	...

Repeat problem 1 for each of these sequences.

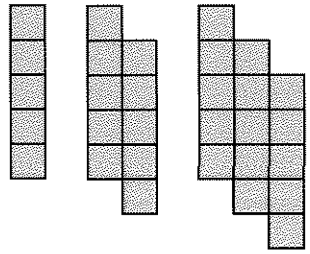
2.



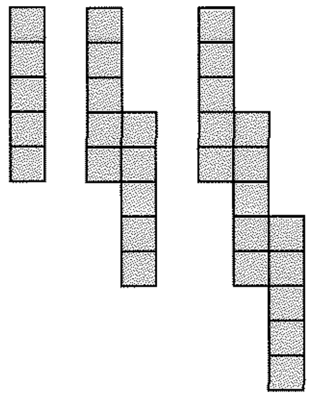
3.



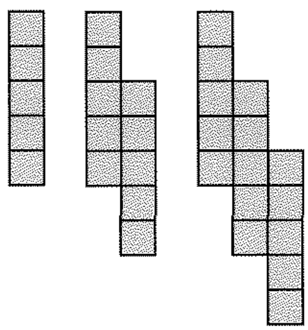
4.



5.



6.



If you have trouble answering questions 7-8 by trial and error, try making graphs from the data in your tables, with the figure number (n) on the horizontal axis and the perimeter on the vertical axis.

7. In problem 1, which figure would have perimeter 50?

8. Is it possible to have perimeter 50 for any of the patterns in problems 2-6?

9. Look at the x-block.
   a. What is the perimeter of its top face?
   b. What is its perimeter if $x = 1, 2, 3, 4, 10$? Make a table like the ones above.
   c. Compare your table with those in problems 1-6. It should be the same as one of them. Which one? Explain why you think this works.

10. a. This figure represents the tops of five x-blocks. What is its perimeter?
    b. What is its perimeter if $x = 1, 2, 3, 4, 10$? Make a table like the ones above.
    c. This figure is related to one of problems 2-6. Which one? Explain.

Note that in problems 9 and 10, just one figure represents a whole infinite sequence of figures, because of the use of variables.

11. Find the blue block that is related to problem 3. Explain.

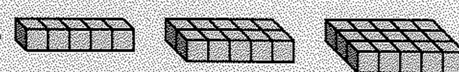
12. For each of problems 4-6, build a related figure made of blue blocks. Check your answer by making a table.

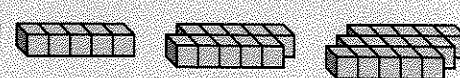
SURFACE AREA

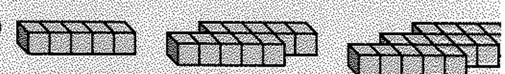
13. Look at the sequence of cube figures. Think about how it would continue, following the pattern. Then:
    a. Sketch the next figure in the sequence.
    b. Copy and complete the following table.
    c. Describe the pattern in words.

Figure #	Surface Area
1	6
2	10
3	14
4	...
10	...
100	...
n	...

Repeat problem 13 for each of these sequences.

14. 

15. 

16. 

17. For each of problems 13-16, build a related figure made of blue blocks. Check your answers by making a table.

MORE SURFACE AREA

18. Look at the sequence. Think about how it continues, following the pattern. Then:
    a. Sketch the next figure.
    b. Make a table like the following one.

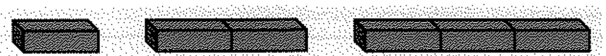
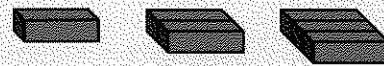


Figure #	Surface Area
1	$4x + 2$
2	$8x + 2$
3	$12x + 2$
4	...
10	...
100	...
n	...

c. Describe the pattern in words.

Repeat problem 18 for each of these sequences.

19. 

20. 

21. 

22. Make a figure out of blue blocks such that by substituting 1, 2, 3, ... for y in its surface area you get the same sequence as you did in problem 19. Check your work by making a table.

GAME *SPROUTS*

This is a game for two players. Start with three dots on a piece of paper. These represent towns. Players take turns. To make a move:

- Join a town to itself or to another town with a *road* (a line).
- Place another town somewhere on the road you just created.

Rules:

- A road cannot cross itself, another road, or an existing town.
- No town can have more than three roads coming out of it.

The winner is the last person able to make a legal move.

23. Play the game with a classmate.

24. What is the maximum number of moves possible in a game?

LESSON 2.11

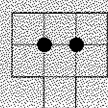
Polyomino Functions

You will need:

graph paper

POLYOMINO EYES

Definition: The points of intersection of the grid lines inside a polyomino are called *eyes.*



1. **Exploration** Any polyomino has an area, a perimeter, and a number of eyes. Is there a relationship between the three numbers? Can you express the perimeter as a function of the area and the number of eyes? (Hint: To find out, draw several polyominoes *that have the same area,* but different perimeters. For each one, write the number of eyes and the perimeter. As the number of eyes increases, does the perimeter get longer or shorter? Repeat the process for a different area.) Write a paragraph telling what you discover.

2. Complete the table shown at the top of the next column. Use data from these figures.

3. Write a formula for the perimeter of a polyomino having area 12 and e eyes.

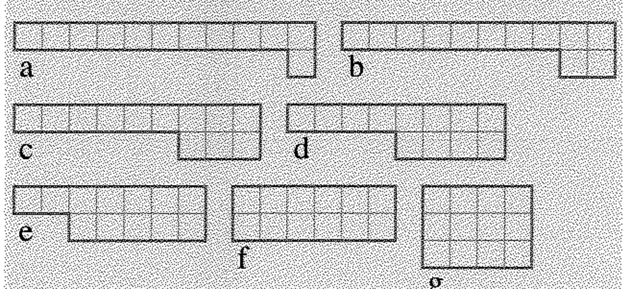


Figure	Eyes	Area	Perimeter
a	0	12	...
b	...	...	...

4. Fill out a similar table for another area. Write a formula for the perimeter as a function of the number of eyes for your area.

5. If you know that a polyomino has 0 eyes and area 100, how could you get its perimeter?

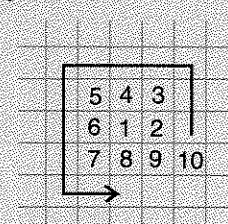
6. Answer question 5 using area 100 and 10 eyes.

7. **Generalization** Write a formula for the perimeter p of a polyomino having area and e eyes. (This formula is a function o two variables, a and e.)

8. For a given area, what is the maximu number of eyes? Find a pattern by exper menting with areas 4 and greater.

A GRAPH PAPER SPIRAL

9. Make a polyomino spiral on your graph paper by shading in one square at a time See the figure below.

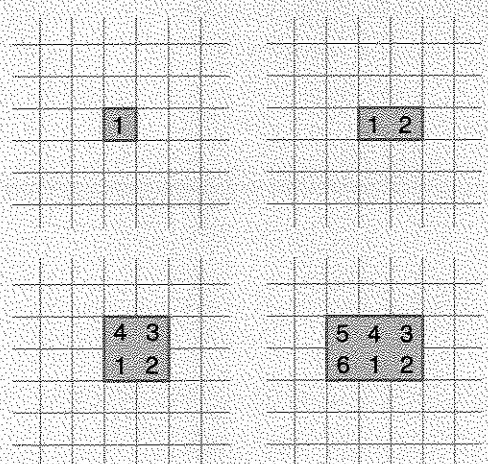


Every time you shade a square, write the perimeter of the figure in a table like the following. Continue until you see a pattern.

Area	Perimeter
1	4
2	6
3	8
...	...

10. Describe the pattern you see.

11. Now make a new spiral the same way. This time record *only* the areas of squares and rectangles that you get along the way, in two tables like those below, continuing until you see a pattern in all the columns.



Square #	Area	Perimeter
1	1	4
2	4	8
3	...	...

Rectangle #	Area	Perimeter
1	2	6
2	6	10
3	...	...

12. Describe the patterns you see in each column.

13. What will the area and perimeter be for square #100?

14. Write a function for:

a. the area of square #*x*;

b. the perimeter of square #*x*.

15. What will the area and perimeter be for rectangle #100?

16. Write a function for:

a. the area of rectangle #*x*;

b. the perimeter of rectangle #*x*.

17. **Report** What do you know about the relationship between area and perimeter of polyominoes? You may draw information from this lesson, as well as from Chapter 1, Lessons 1 and 2. Use graphs and illustrations.

LESSON 2.12

Geoboard Triangles

You will need:
- geoboards
- dot paper

1. **Exploration** If many triangles have one vertical side in common, how is their area related to the position of the third vertex? To find out, make many triangles having vertices at (0, 0) and (0, 8). For each one, keep a record of the coordinates of the third vertex and the area. Look for patterns. Write a paragraph explaining what you found out. Use sketches.

HORIZONTAL *AND* VERTICAL SIDES

2. Make a triangle having a horizontal side of length 6 and a vertical side of length 4. What is its area?

3. In this problem, use triangles having a horizontal side of 6.
   a. Make a table like the following. All triangles should have a horizontal side of length 6, but the length of the vertical side will vary. Extend the table all the way to vertical side of length 10.

Vertical Side	Area
0	...
1	...
2	6
...	...

   b. Explain how you could find the area (a triangle having horizontal side 6 anc vertical side 100.
   c. Express the area as a function of the vertical side.

4. Repeat problem 3 for a horizontal side o: length 9.

ONE HORIZONTAL *OR* VERTICAL SIDE

5. Make a triangle having vertices at (0, 0) and (0, 7) and the third vertex at (1, 4). What is its area?

6. Make a table like the following for triangles having vertices at (0, 0) and (0, 7) a: the third vertex as indicated. Extend the table all the way to vertex (7, 4).

3rd Vertex	Area
(0, 4)	...
(1, 4)	...
(2, 4)	...
...	...

7. Write the area as a function of the *x*-coo: dinate of the third vertex.

8. a. Make the triangle having vertices (0, ((0, 7), and (9, 4). Guess its area.
   b. With another rubber band, make the smallest rectangle that covers the tria: gle. If you did it correctly, you should now see two new triangles. Find the area of the rectangle and the area of tl two new triangles.
   c. Find the area of the original triangle. This should match your guess from part (a).

9. How would you find the area of the triangle having vertices at (1, 0), (6, 0), and (9, 9)? Find it and explain what you did, using a sketch and a paragraph.

Generalizations

10. a. Make triangles having vertices at (0, 0) and (0, 6) and the third vertex at $(x, 9)$, where x takes each of the whole number values from 0 to 10. Make a table of values to show the area as a function of x.
    b. Make triangles having vertices at (0, 0) and (0, 6) and the third vertex at $(9, y)$, where y takes each of the whole number values from 0 to 10. Make a table of values to show the area as a function of y.
    c. How do the answers to (a) and (b) differ?

11. a. Make at least three triangles having vertices at (0, 1) and (0, 6) and the third vertex at (x, y), where x and y take whole number values from 1 to 8. Sketch each one and find its area.
    b. Explain how you would find the area of a triangle having vertices at (0, 1), (0, 6), and (99, 99) without drawing a picture.

12. Explain how you would find the area of a triangle having vertices at (0, 0), $(b, 0)$, and (x, h), where b and h are nonnegative.

NO HORIZONTAL OR VERTICAL SIDES

13. **Exploration** What is the area of the triangle having vertices (0, 6), (7, 8), and (6, 1)? Explain how you arrive at the answer. Use sketches on dot paper.

14. What is the area of the four-sided shape having vertices at (0, 7), (2, 10), (10, 5), (5, 0)? Hint: First find the area of the whole geoboard, then use subtraction.

15. Make a triangle having no horizontal or vertical sides and having vertices on the outside edges of the geoboard. Use subtraction to find its area.

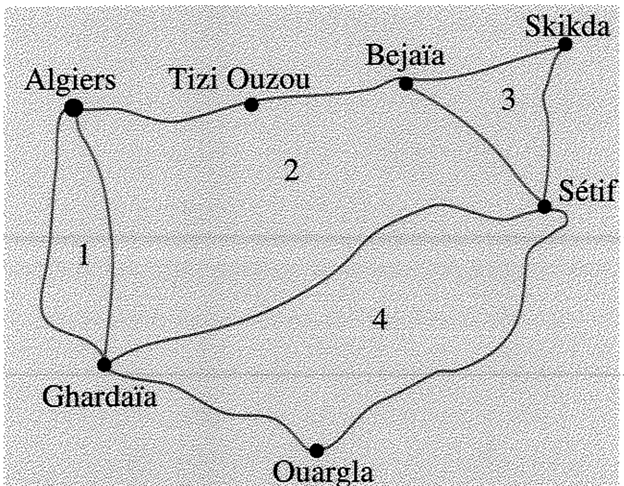
16. Repeat problem 15 on another triangle.

17. What is the area of the triangle having vertices at (1, 8), (2, 4), and (9, 3)? Hint: You may use the triangles having these vertices.

    (1, 8), (1, 3), (9, 3)
    (2, 4), (1, 3), (9, 3)
    (1, 8), (2, 4), (1, 3)

18. **Report** Write an illustrated report on how to find the area of any geoboard triangle. Give examples of the different techniques. Make sure you include examples of using division by two, addition, and subtraction.

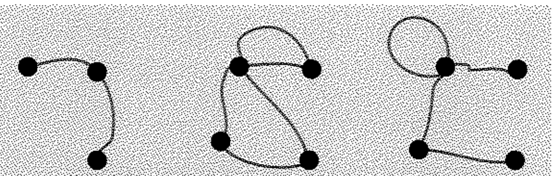
THINKING WRITING 2.C Towns, Roads, and Zones



This is a simplified road map of part of Algeria. It shows 7 towns and 10 roads. For the purposes of this lesson we will call any area completely surrounded by roads, (and not crossed by any road,) a *zone*. As you can see, there are 4 zones on this map.

Rules: Each town is connected to all the others by roads (not necessarily a direct connection); all roads begin and end at a town. It is possible for a road to connect a town to itself. It is possible for more than one road to connect two towns.

In maps like this one there is a relationship between the number of towns, roads, and zones. Your goal in this lesson is to find it. The relationship was discovered by the Swiss mathematician and astronomer Leonhard Euler. It is part of a branch of geometry called *topology*, which he created.



1. **Exploration** Make many different "maps like the ones above. Keep track of the number of roads, towns, and zones in a table. Try to find a pattern in the relatioi ship of the three numbers. (If you canno find a relationship between all three nun bers, keep one of the numbers constant and look for a relationship between the other two.)

2. Make at least six different three-town maps. What is the relationship between number of roads and the number of zone Express it in words, and write r (the nur ber of roads) as a function of z (the num ber of zones).

3. Make at least six different four-town maps. What is the relationship between number of roads and the number of zone Express it in words and write a function

4. Make at least six different five-road ma What is the relationship between the nu ber of towns and the number of zones? Express it in words and write a function

5. Make at least six different six-road maps. What is the relationship between the number of towns and the number of zones? Express it in words and write a function.

6. Make at least six different four-zone maps. What is the relationship between the number of roads and the number of towns? Express it in words and write a function.

7. **Report** Write an illustrated report describing what you have learned about towns, roads, and zones. Give examples. Your report should answer the following questions, but not be limited to them:
   - If there are t towns and r roads, how many zones are there?
   - If there are t towns and z zones, how many roads are there?
   - If there are r roads and z zones, how many towns are there?

8. **Project** **Euler**
   Find out about Leonhard Euler and/or the Koenigsberg Bridge Problem. Prepare an oral presentation or a bulletin board display.

Essential Ideas

THREE MEANINGS OF MINUS

1. For each of the following, write an explanation of what the minus sign means.
   a. -2 b. $-(2 + 2x)$
   c. $x - 2$ d. $-y$

OPPOSITES

2. Find the opposite of each quantity. Remember: A quantity and its opposite add up to zero.
   a. x b. 2
   c. -2 d. $-x$
   e. $x + 2$ f. $x - 2$

ADDING AND SUBTRACTING

In problems 3-4 you may want to make sketches or use the Lab Gear.

3. Simplify. (Add and combine like terms.)
   a. $(y^2 + x^2 - 3y) + (y + 3x^2 - x^2)$
   b. $x + (25 - yx - y^2) + (xy - y - x)$
4. Simplify. (Subtract; combine like terms.)
   a. $(4 - x^2 - 5x) - 3x - 2$
   b. $(4 - x^2 + 5x) - (3x - 2)$
   c. $(4 + x^2 - 5x) - (3x + 2)$
   d. $(-4 - x^2 - 5x) - (-3x + 2)$

MULTIPLYING

In problems 5-8 you may want to make sketches or use the Lab Gear.

5. Multiply.
   a. $2x \cdot 4x$ b. $5x \cdot 6y$
   c. $3xy \cdot 10$
6. The quantity $36xy$ can be written as the product $9x \cdot 4y$. Write $36xy$ as a product in at least four other ways.
7. Multiply.
   a. $2(x + y - 5)$ b. $x(x + y + 5)$
   c. $x(-x + y + 5)$
8. Choose two of the three multiplications in problem 7. Make a sketch of what they look like when modeled with the Lab Gear.

EXPONENTIAL NOTATION

9. Write each of these numbers in exponential notation. If possible, find more than one way. It may help to use your calculator.
   a. 32 b. 64 c. 256
   d. 4096 e. 1 f. 6561

FUNCTIONS AND FUNCTION DIAGRAMS

For each of the following problems:

a. Copy the table.
b. Describe the rule that allows you to g y from x.
c. Use the rule to find the missing numbers. (In some cases, the missing numbers may be difficult to find; use trial and error and a calculator to mak it easier.)
d. Write y as a function of x.

10.

x	y
-1	-7
4	28
0	
	7

11.

x	y
3	4
12	1
6	2
	5

12.

x	y
5	2
	4
1	
	-1

13. a. Make a function diagram in which th output (y) is always 4 more than the input (x).
    b. Write a rule (function) for your funct diagram.

14. a. Make a function diagram in which the output (y) is always 4 times the input (x).
b. Write a rule (function) for your function diagram.

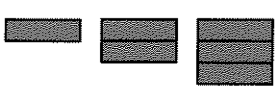
15. Make a function diagram with *time* on the x-number line (show one hour from the bottom to the top), and *distance* on the y-number line, to represent the motion of a cyclist riding at a constant speed of 15 miles per hour. Your diagram should have five in-out lines.

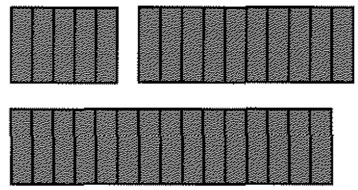
PATTERNS AND FUNCTIONS

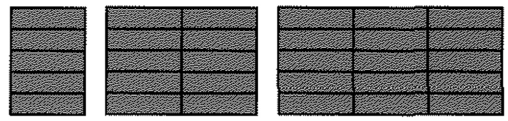
16. Look at the sequence of figures. Think about how it would continue, following the pattern. Then:
a. Sketch the next figure in the sequence.
b. Copy and complete a table like the one below.
c. Describe the pattern in words.

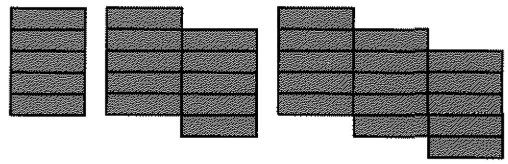
Figure #	Perimeter
1	...
2	...
3	...
4	...
10	...
100	...
n	...

Repeat problem 16 for these sequences.

17. 

18. 

19. 

20. 

21. In problem 16, what figure would have a perimeter of $88x + 2$? Use trial and error if necessary.

22. Which sequence in problems 17-20, if any, contains a perimeter of
a. $2x + 100$?
b. $100x + 2$?
c. $100x + 100$?

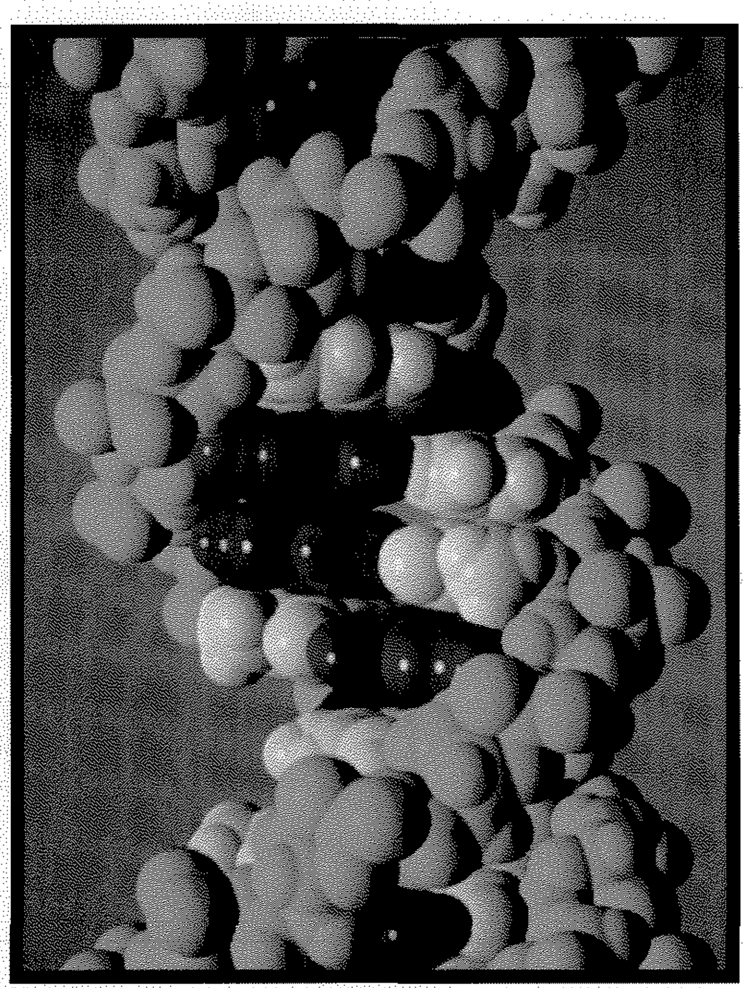
23. Look at the xy-block.
a. What is the perimeter of its top face?
b. What is its perimeter if $y = 1, 2, 3, 4, 10$? (Do not substitute a number for x.) Arrange your answers in a table.
c. Compare your table with those in problems 16-20. It should be the same as one of them. Which one? Explain.

24. Use blue blocks to make a figure. Substitute 1, 2, 3, ... for y in its perimeter to get the same sequence as problem 18. Check your work; make a table.

GEOBOARD TRIANGLES

25. On dot paper, sketch triangles having area 18, and having
a. one horizontal and one vertical side;
b. one horizontal side, no vertical side;
c. no horizontal or vertical side.

CHAPTER 3



The double helix of a DNA molecule

Coming in this chapter:

Exploration Algebank offers to double your money every month, in exchange for a monthly fee. Is this a good deal? Does the answer depend on the fee, on the amount of money you have to invest, or on both?